Heterogeneous Differential Privacy via Graphs

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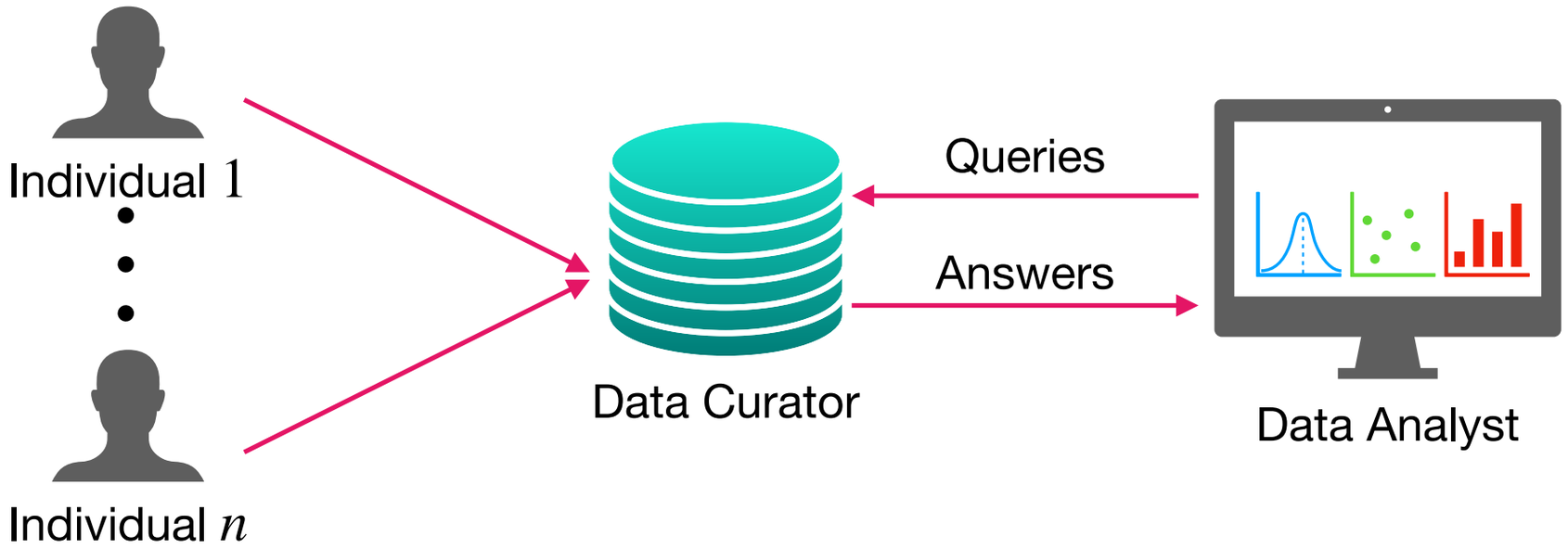
Private Data Analysis



- Individuals share private data with a curator.
- The Curator manages the release of the data to Analysts.
- Two conflicting goals:
  - **Privacy:** protecting the individuals' privacy.
  - **Utility:** learning useful information about the population.

What is **Differential Privacy?**

- First proposed in 2006 **[Dwork et al. '06]**.
- **Goal:** To learn as little as possible about an individual while learning useful information about a population.
- **Idea:** To add a controlled amount of randomness to the dataset.
- **Highlighted applications:**
  - Google, for sharing historical traffic statistics **[Erlingsson et al. '14]**.
  - Apple's private learning of users' preferences **[Apple DP team '17]**.
  - Microsoft for telemetry in Windows **[Ding et al. ’17]**.
  - The 2020 United States Census.

Some Issues with Traditional Differential Privacy

- A “one-size-fits-all" approach to setting a global privacy level can be damaging to both utility and privacy. **[Jorgensen et al. '15]**

- Some groups might need more protection than others. **[Kohli et al. '18]**

- There may also be statutory mandates, demanding publication of certain datasets with more accuracy. (e.g. US Census)

Main Contributions

- The notion of **Heterogeneous Differential Privacy**.

- Characterizing **optimal** schemes for binary functions.

- Presenting a **low complexity algorithm** for finding such schemes.

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- A symmetric relationship in $\mathscr{D}$ where $d \sim d'$ are said to be neighbors.

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$\mathscr{M}$ is ε-differentially private if, for any $d \sim d'$ and $\mathcal{S} \subseteq \mathscr{D}$,

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Traditional Differential Privacy

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- An output space $\mathscr{Q}$. **We consider binary $\mathscr{Q} = \{1,2\}$.**
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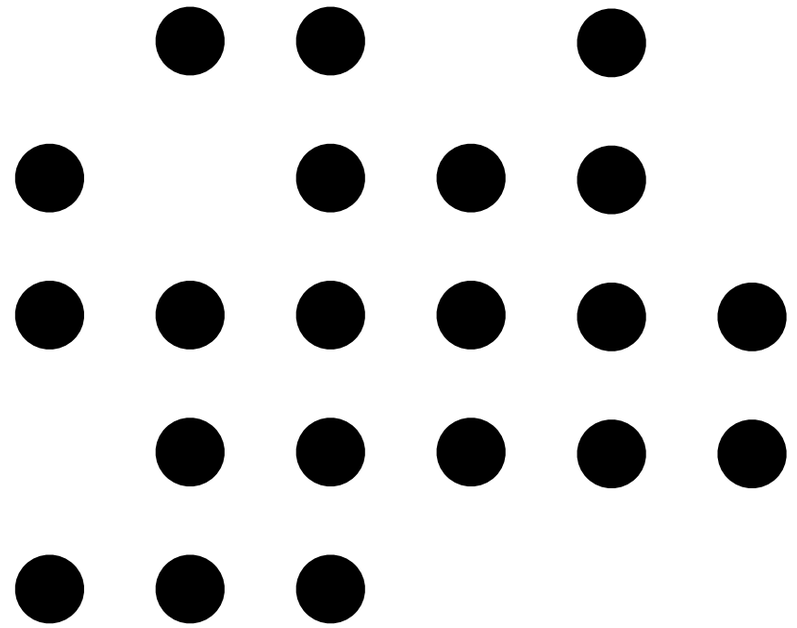

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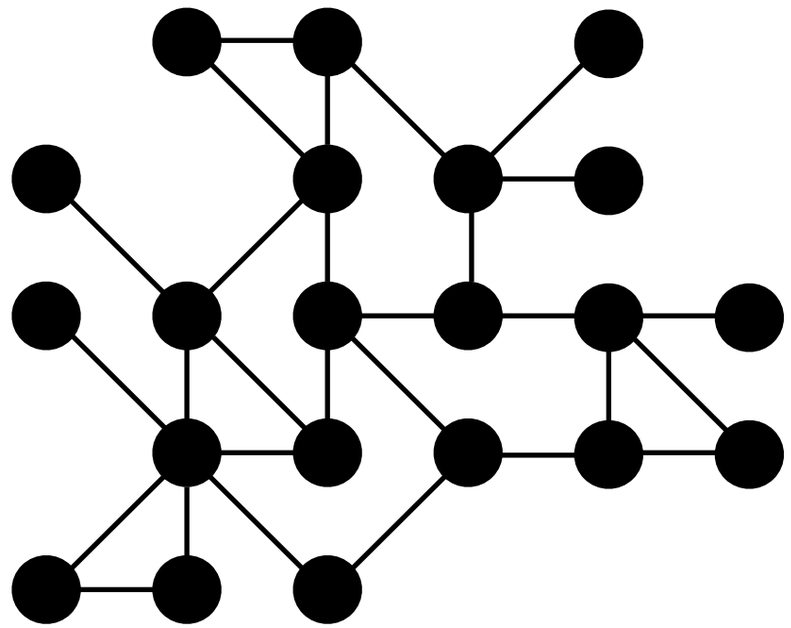

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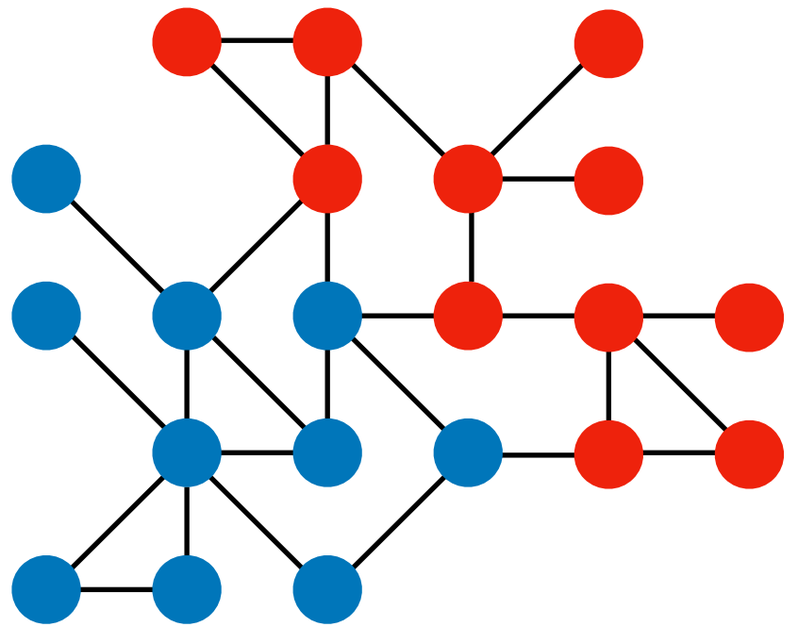

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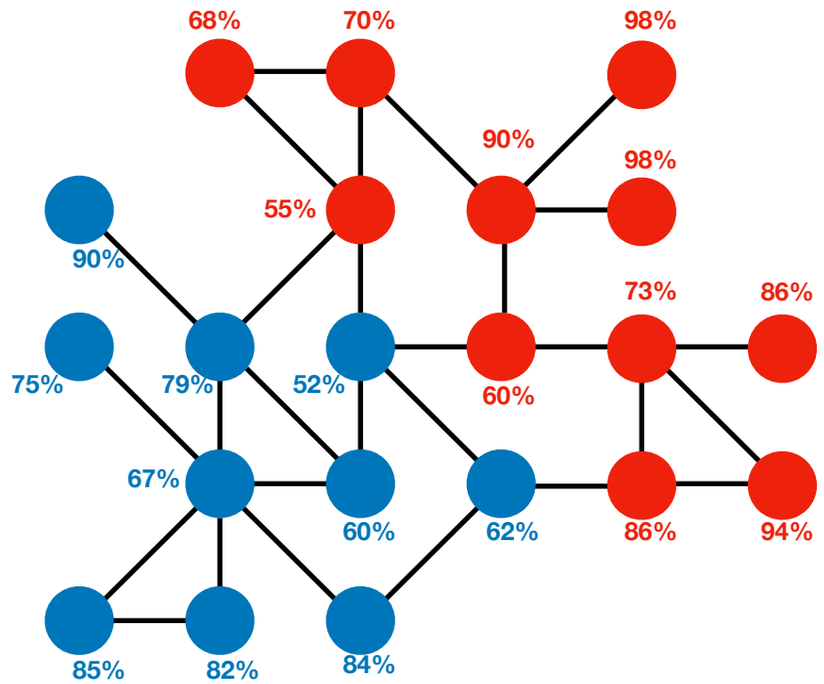

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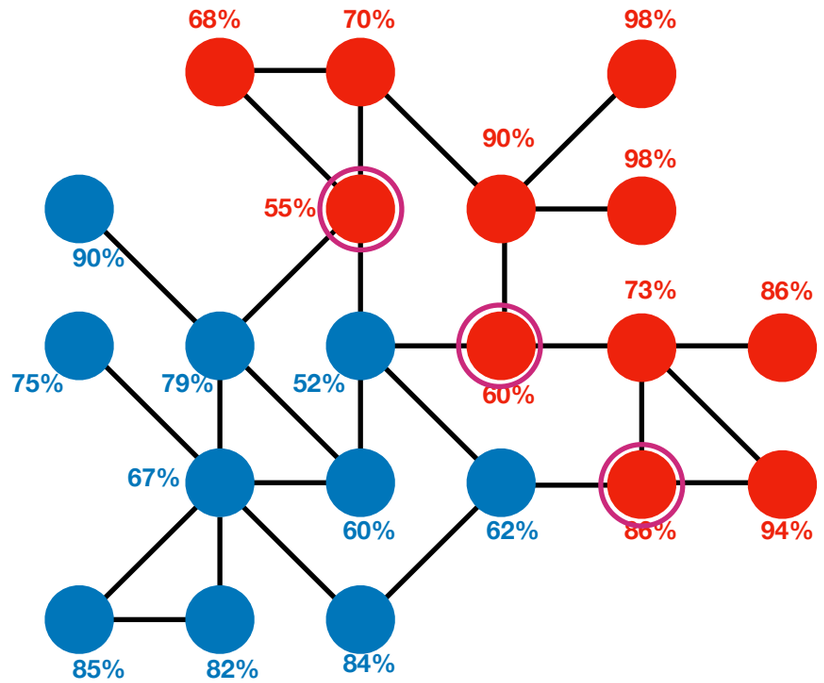

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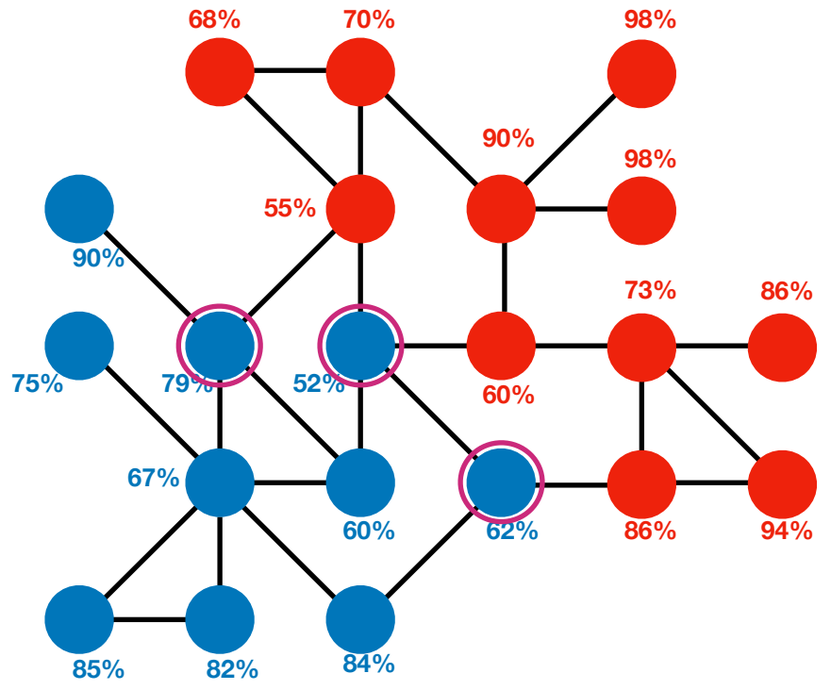

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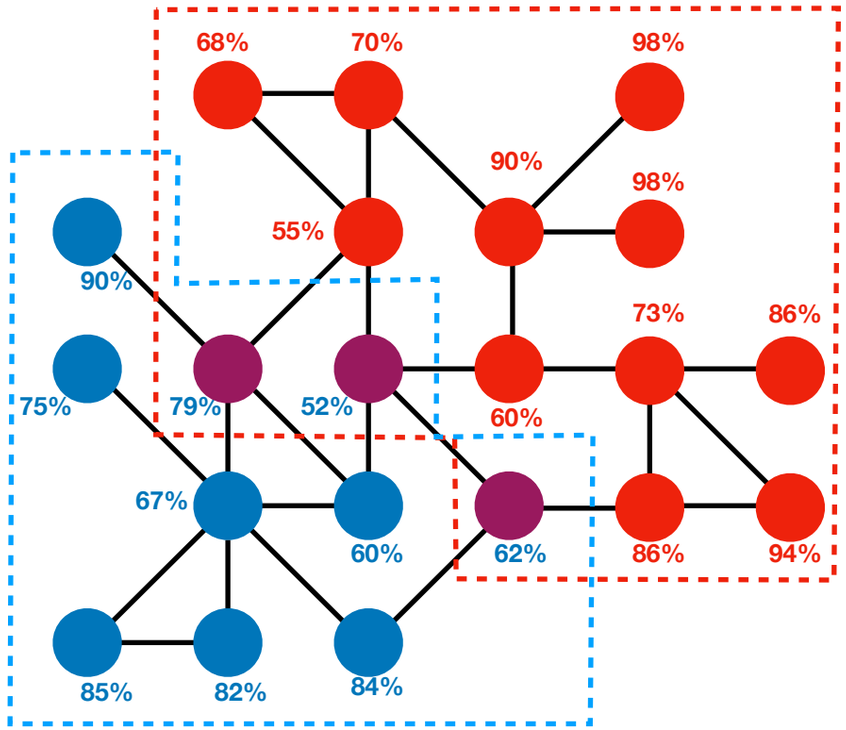

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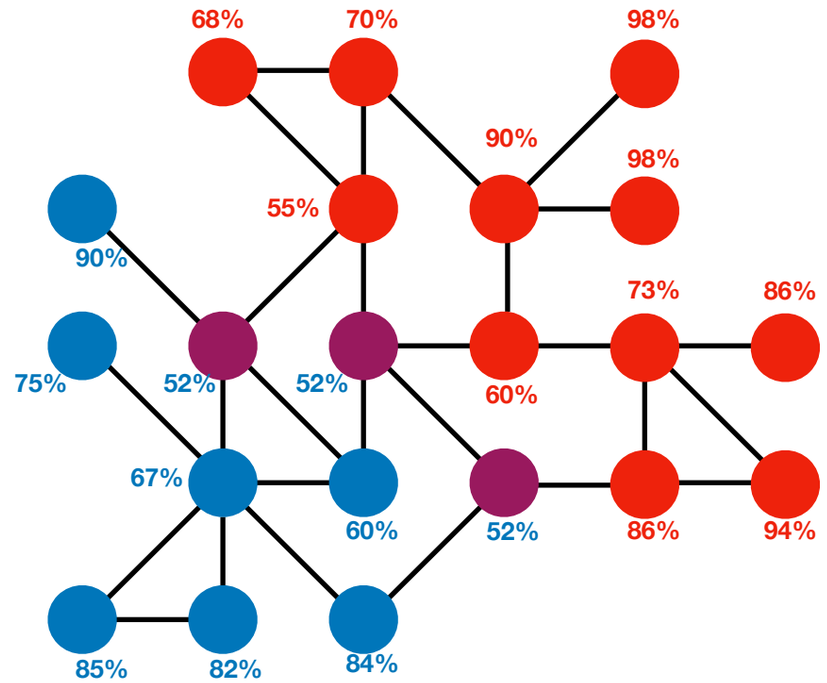

Definition: Boundary Homogeneous

A mechanism $\mathscr{M}$ is boundary homogeneous if the probabilities at the boundary are the same.

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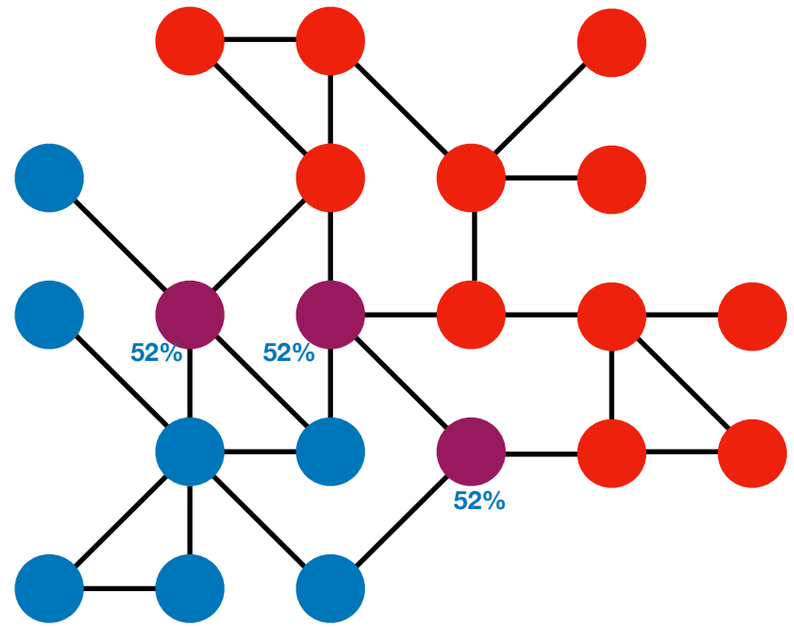
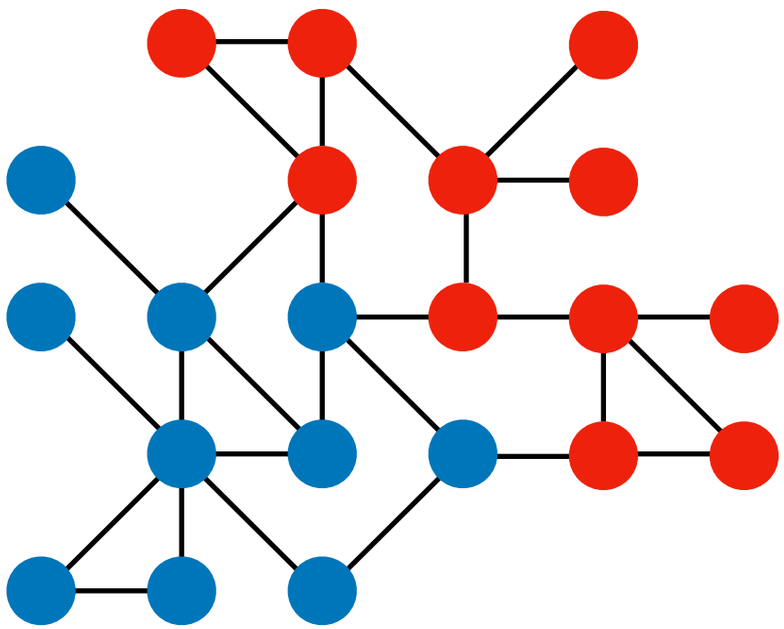


Theorem [D'Oliveira et al. 21']

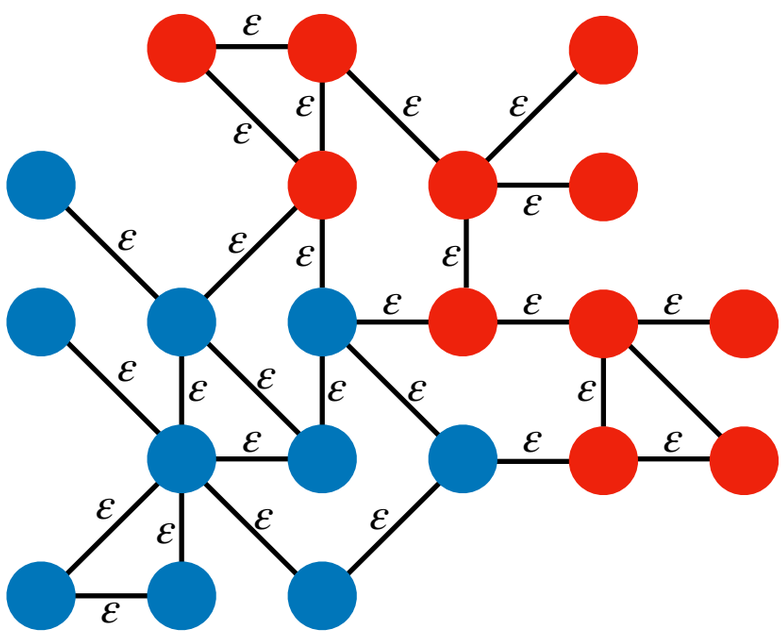
Let the boundary probabilities be equal. Then, there exists at most one optimal ε-DP mechanism. (A closed form is presented in the paper.)

Homogeneous Differential Privacy



Homogeneous Differential Privacy

- Same level of privacy.



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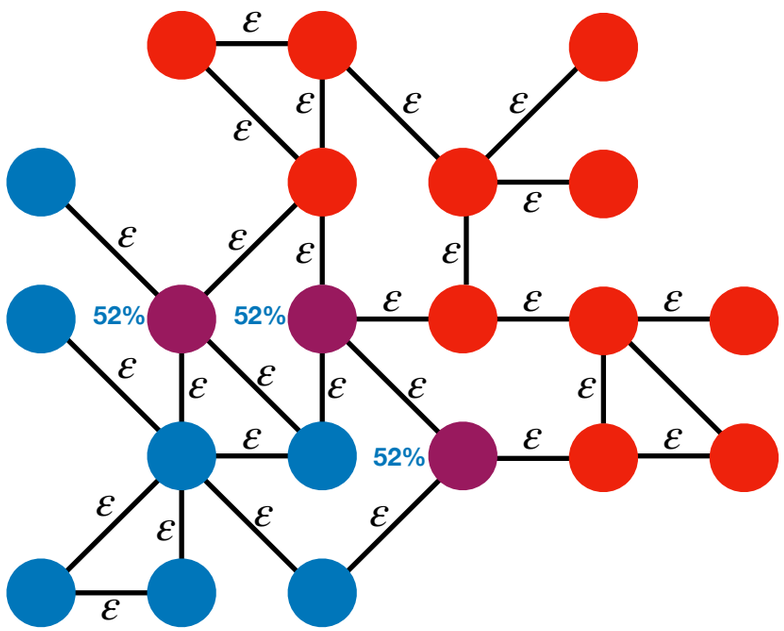

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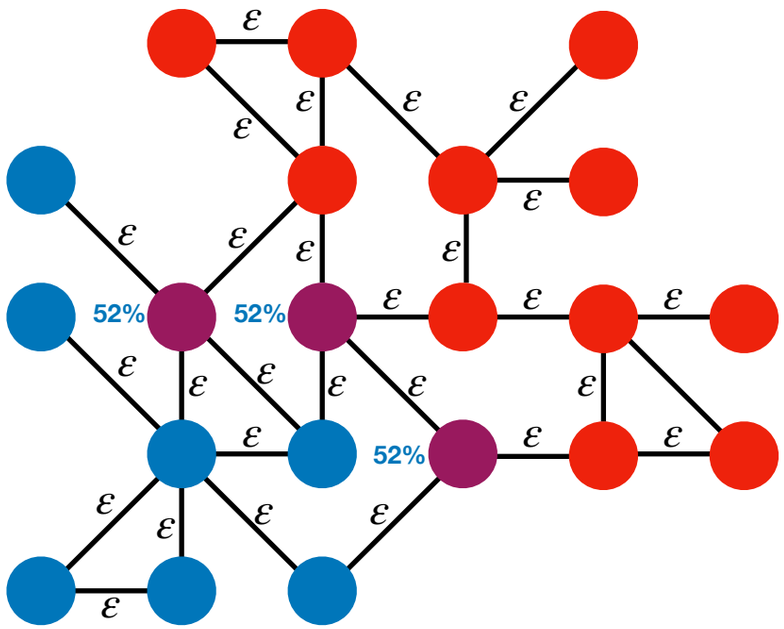

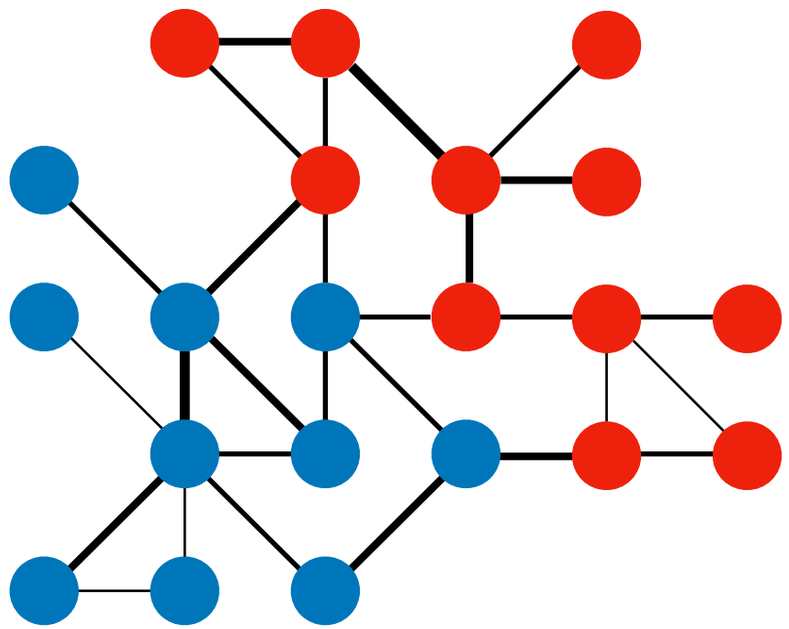

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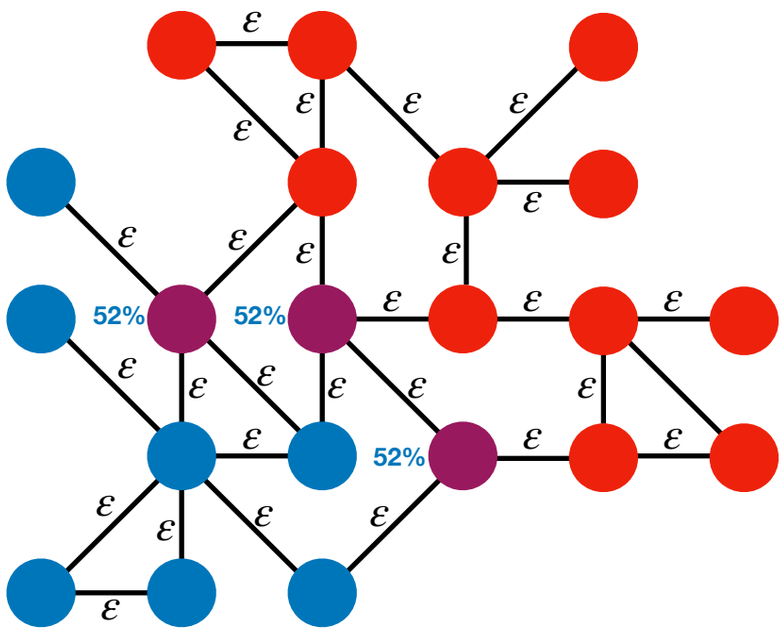

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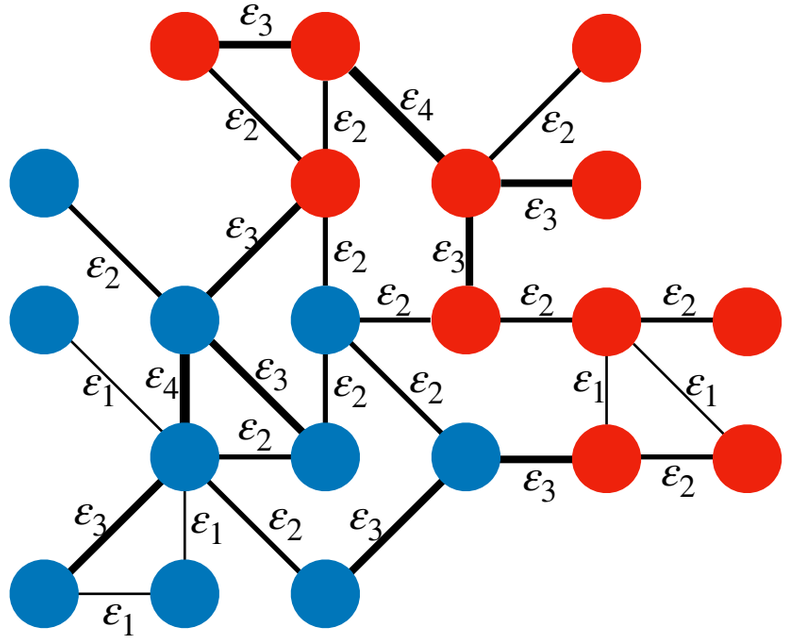

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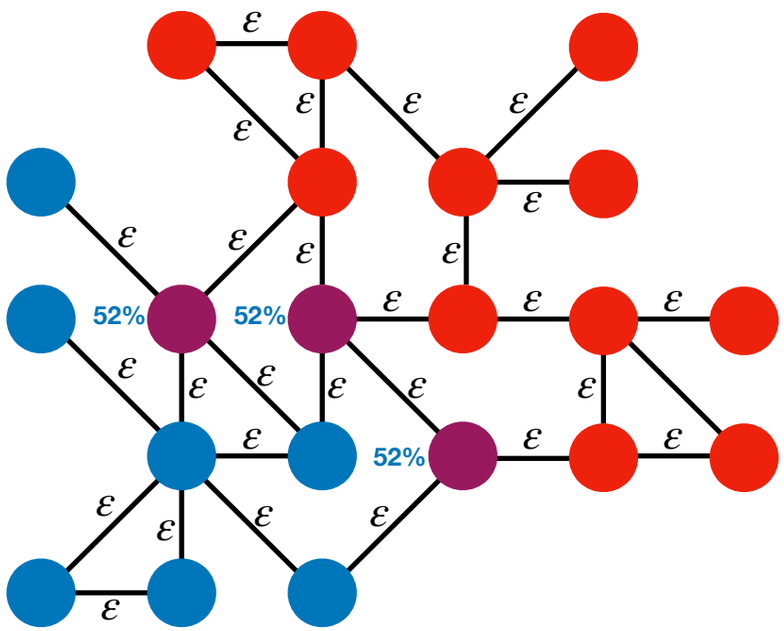
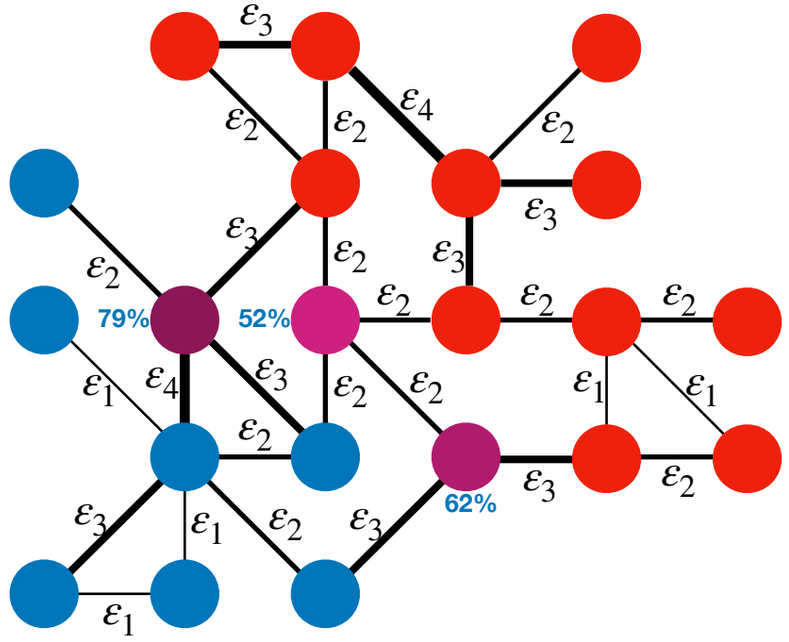
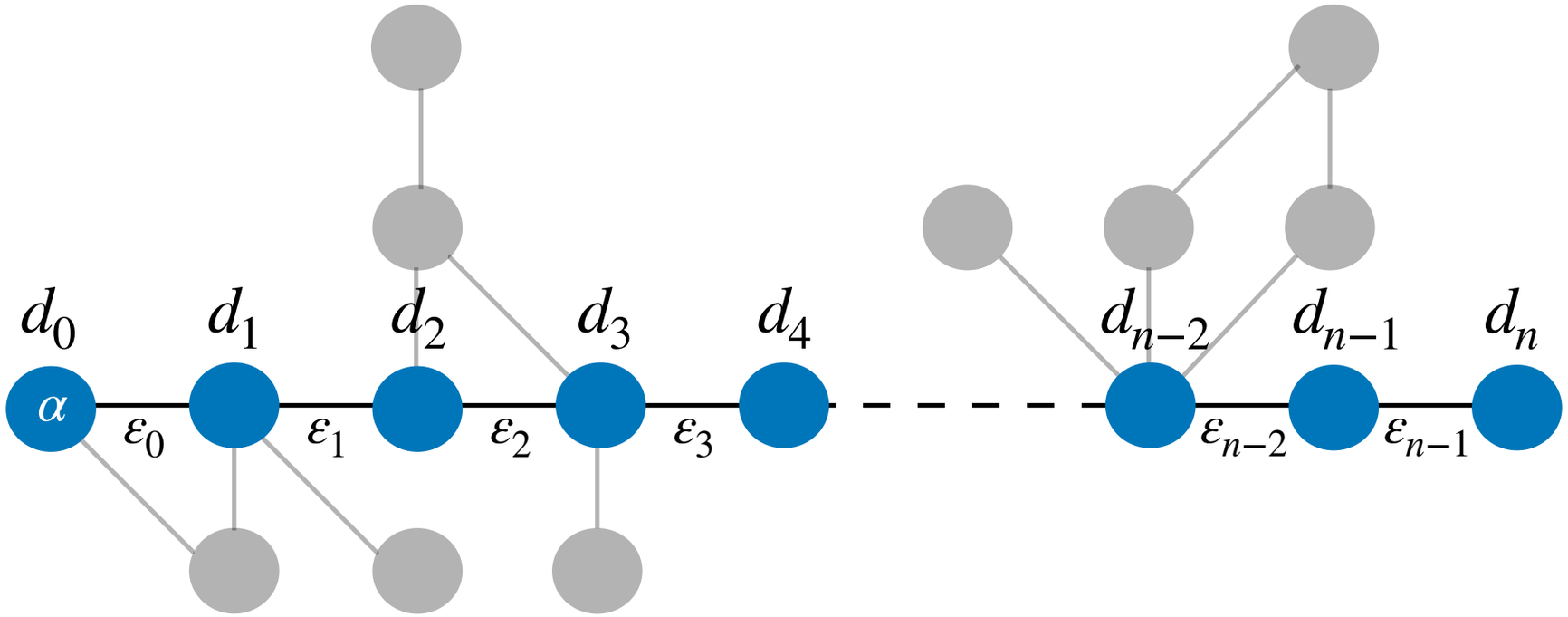
- Different privacy levels.
- Different probability distributions at the boundary





Special Case: Path Graph



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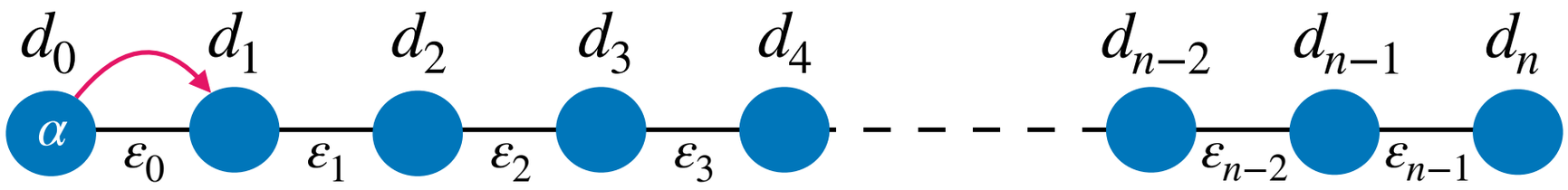

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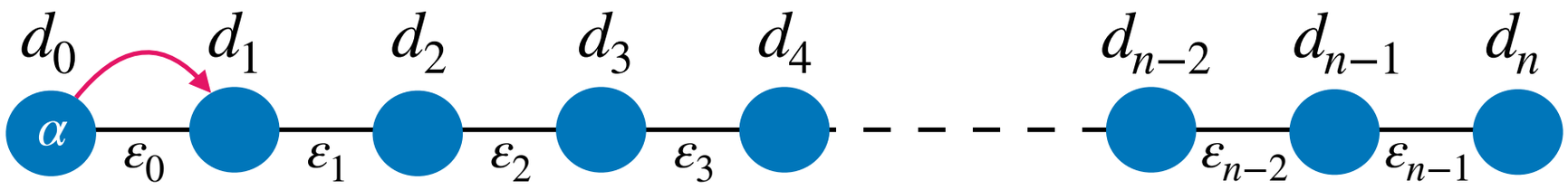

Differential Privacy conditions between d_0 and d_1:

- $\Pr[\mathscr{M}(d_1) = 1] \leq e^{\varepsilon} \Pr[\mathscr{M}(d_0) = 1]$
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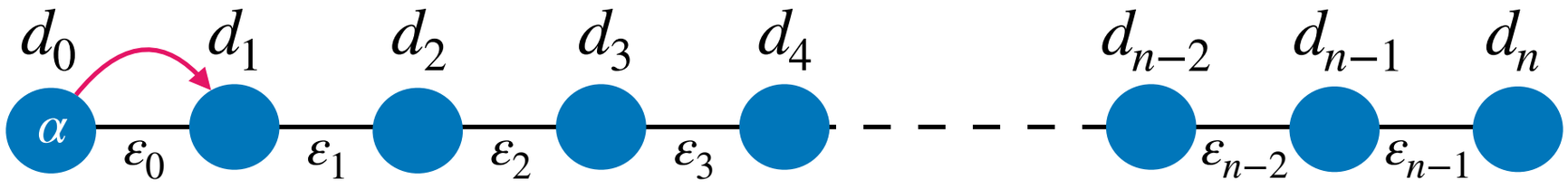

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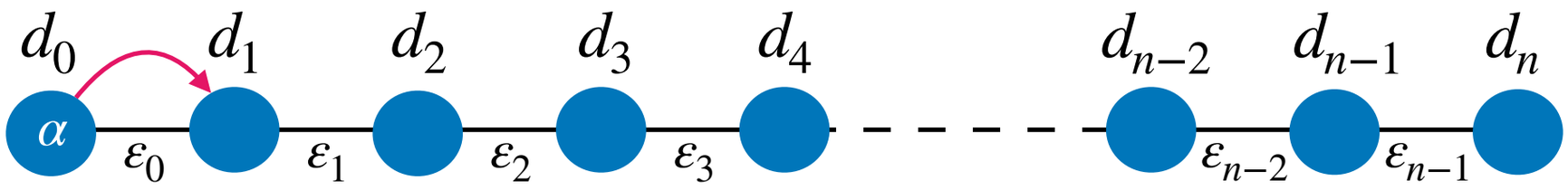

Upper bound on d_1 imposed by d_0:

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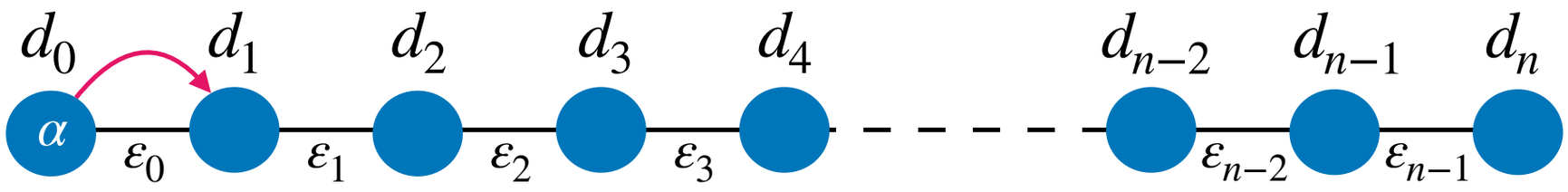

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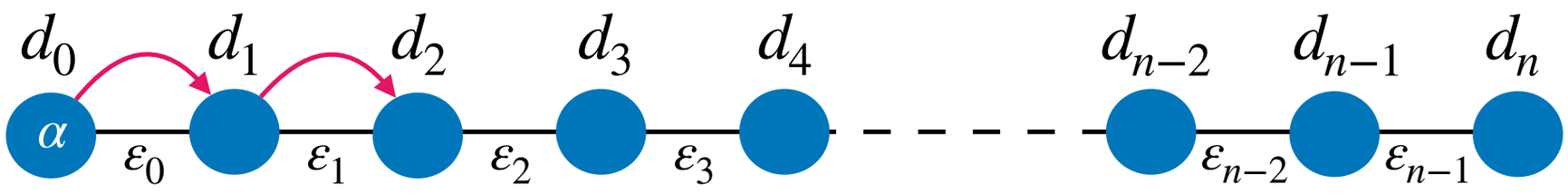

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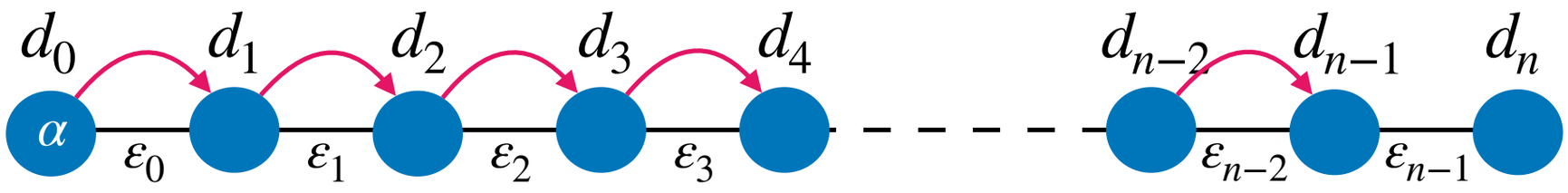

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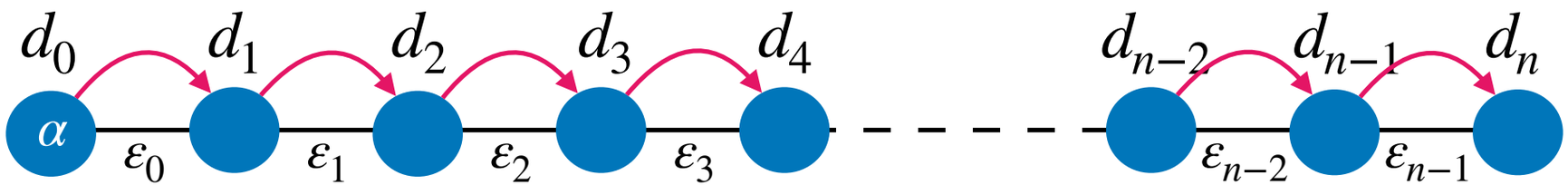

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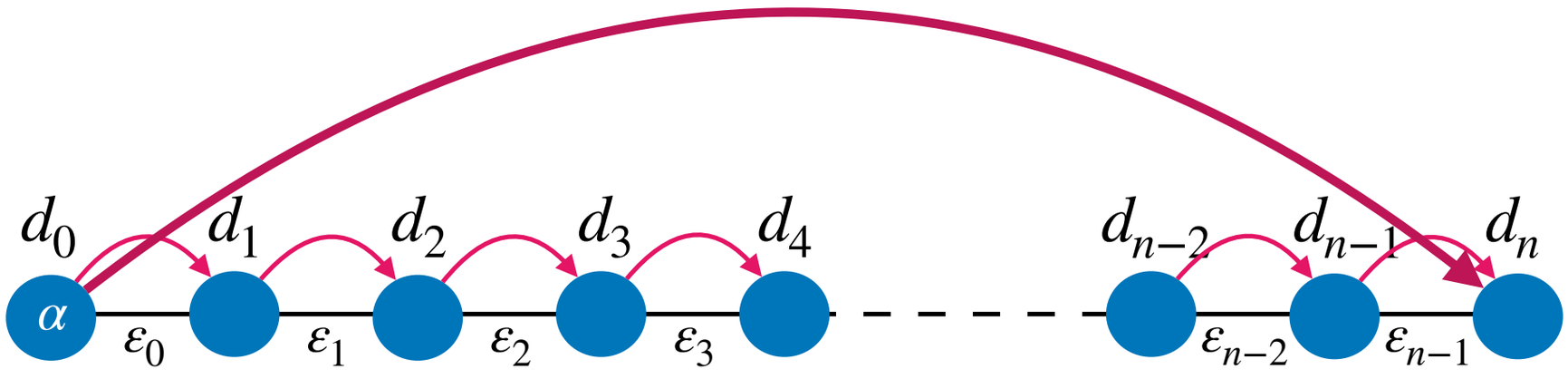

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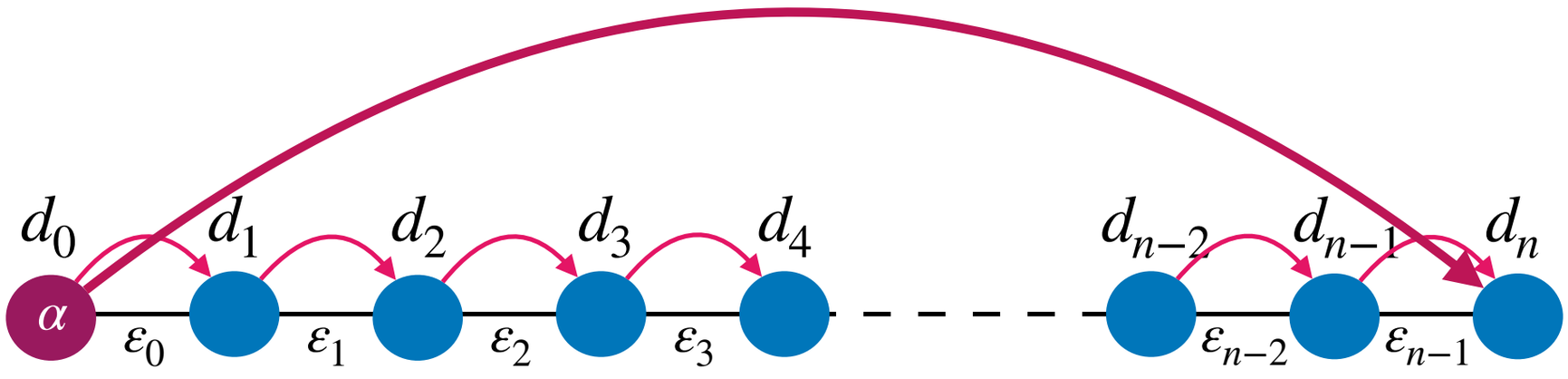


Lemma: Differential Privacy for the Path Graph

The optimal binary-valued heterogeneous differentially private mechanism $\mathscr{M}^*$ is given by

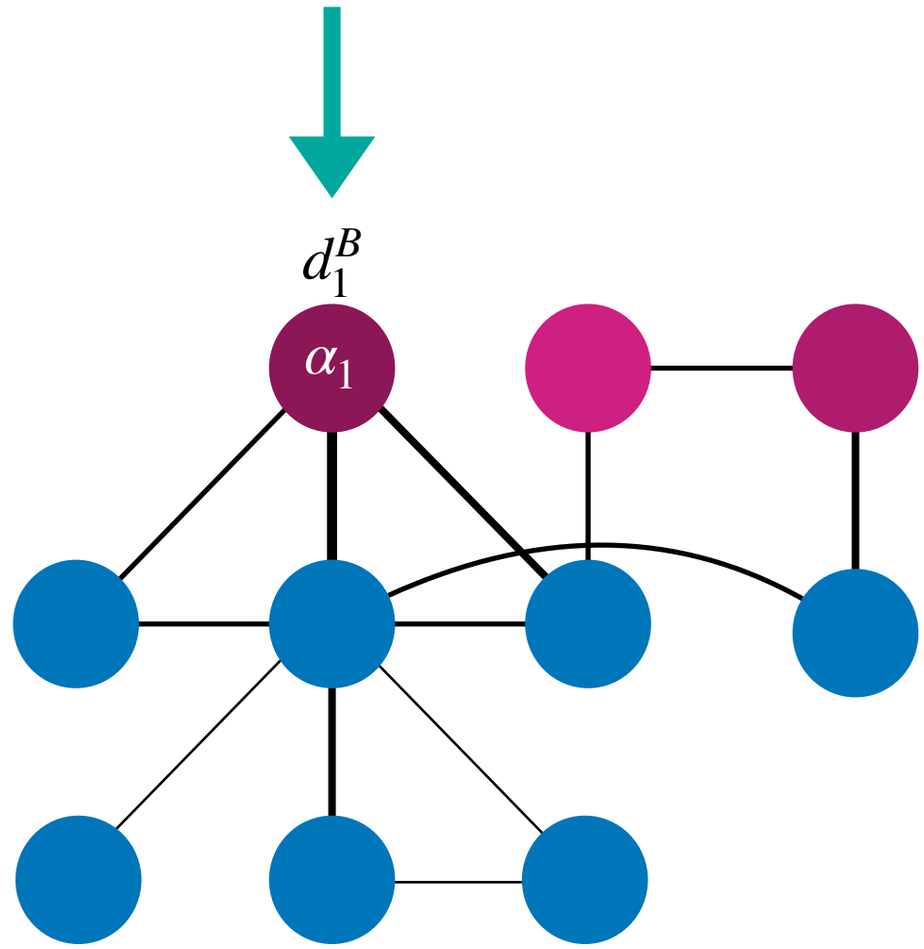
$$\Pr[\mathscr{M}^*(v_i) = 1] = \begin{cases} e^{\varepsilon_{i-1}+\ldots+\varepsilon_1+\varepsilon_0}\,\alpha & i \le \tau \\ e^{-\varepsilon_{i-1}-\ldots-\varepsilon_1-\varepsilon_\tau+\varepsilon_{\tau-1}+\ldots+\varepsilon_1+\varepsilon_0}\,\alpha + 1 - e^{-\varepsilon_{i-1}-\ldots-\varepsilon_1-\varepsilon_\tau} & i > \tau \end{cases}$$

where

$$\tau = \arg\min_{i\in[n]} \{\frac{1}{\alpha} \le e^{\varepsilon_{i-1}+\ldots+\varepsilon_1+\varepsilon_0}(e^{\varepsilon_i}+1)\}\,.$$

Algorithm

- For each boundary vertex d_i^B with a fixed value α_i, the upper bound on the other vertices imposed by d_i^B is calculated in the following steps.



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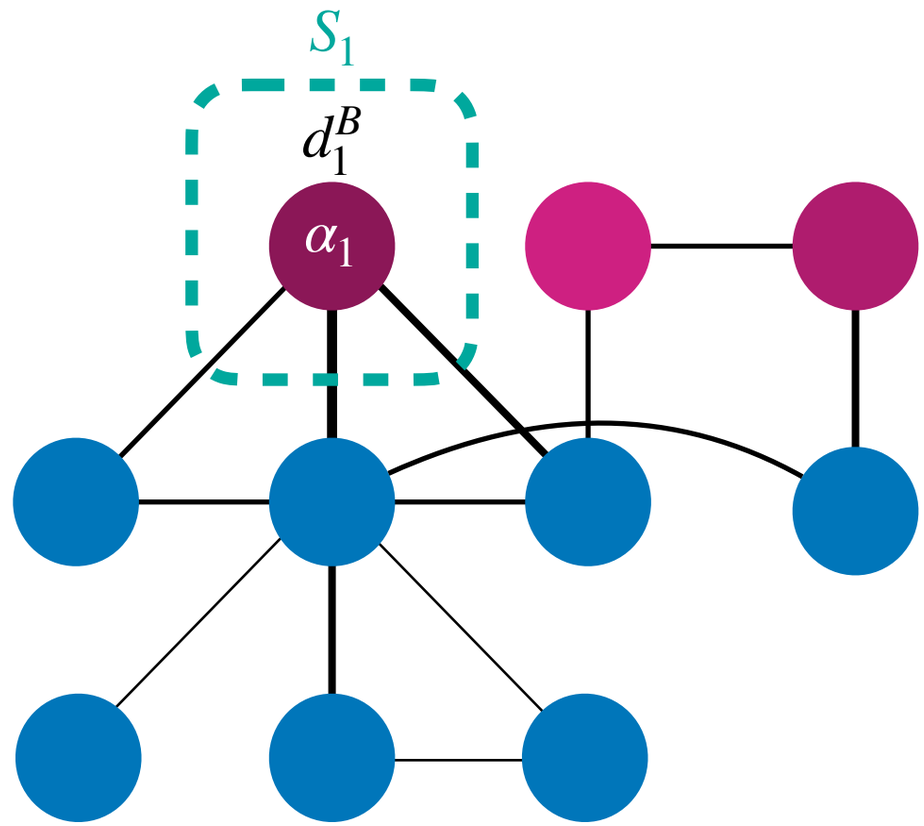

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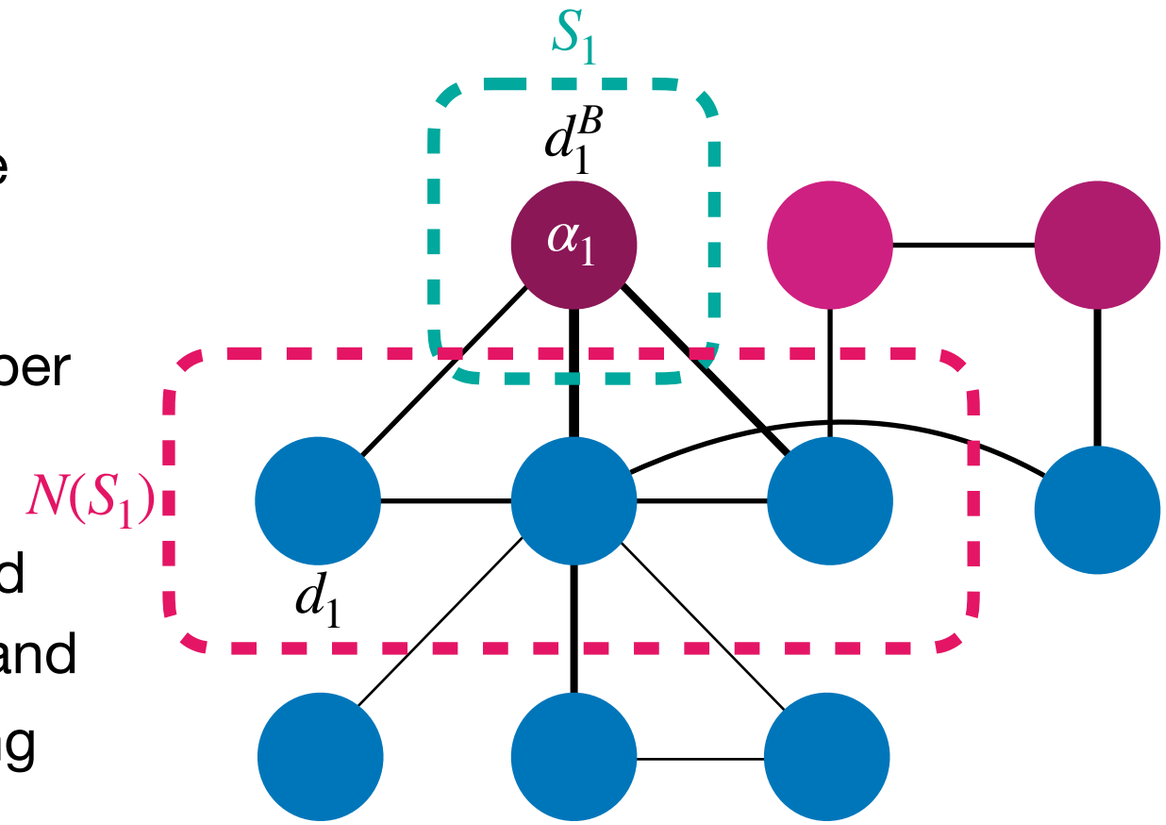

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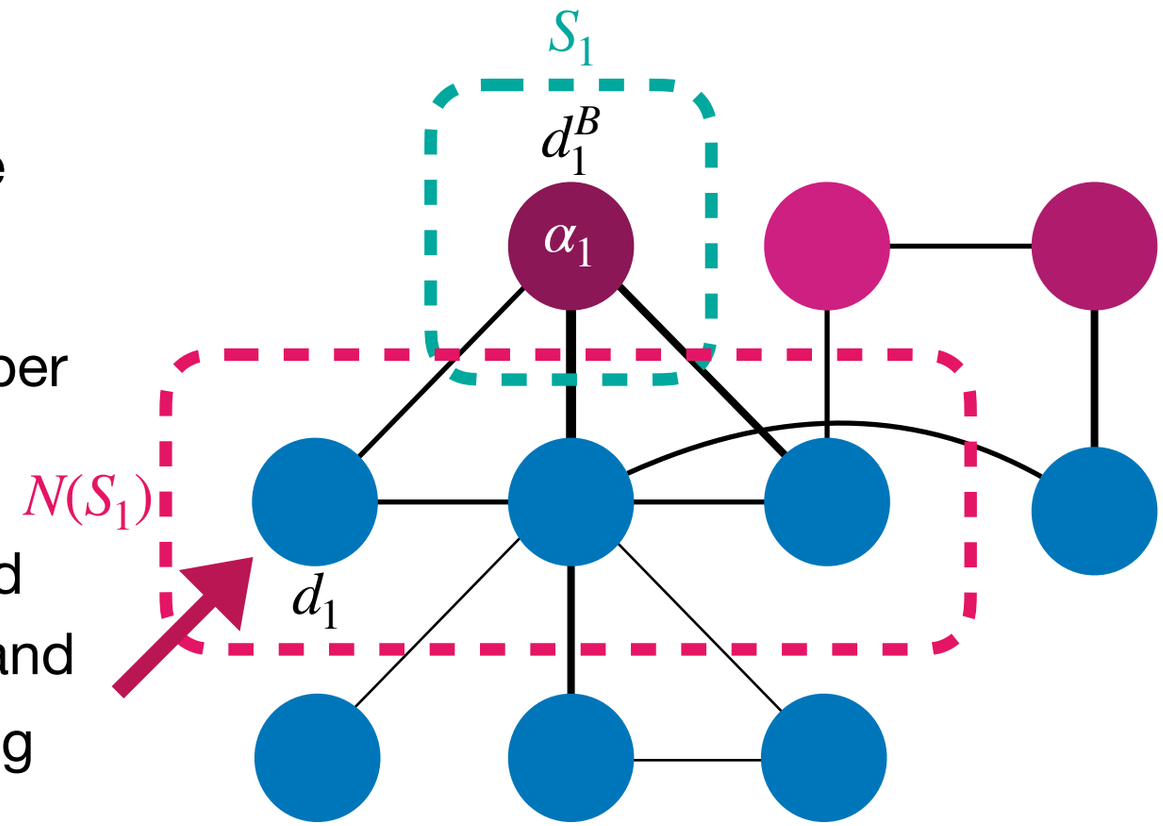

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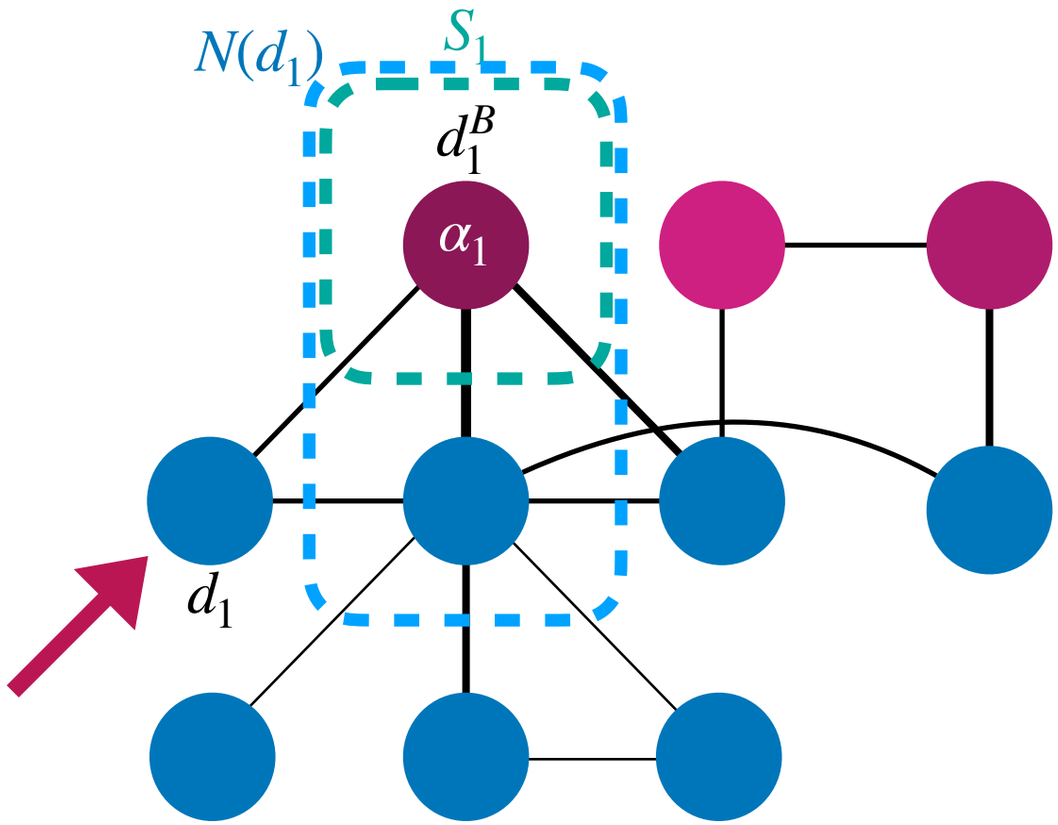

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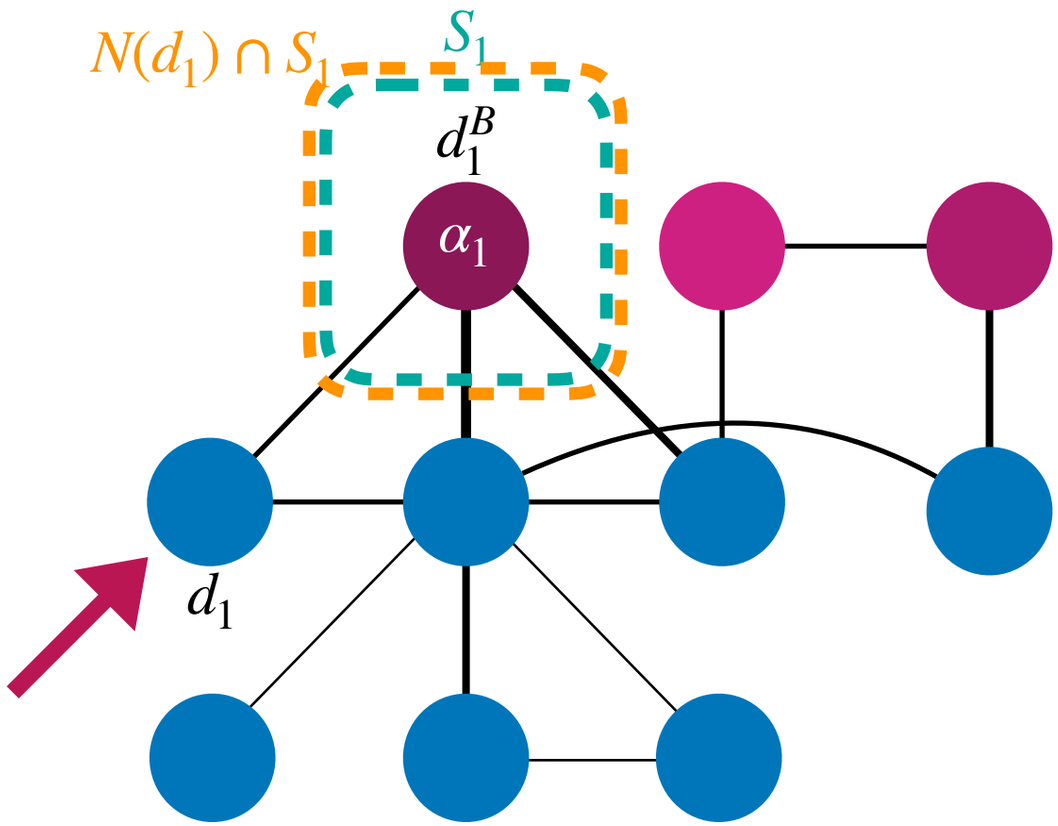

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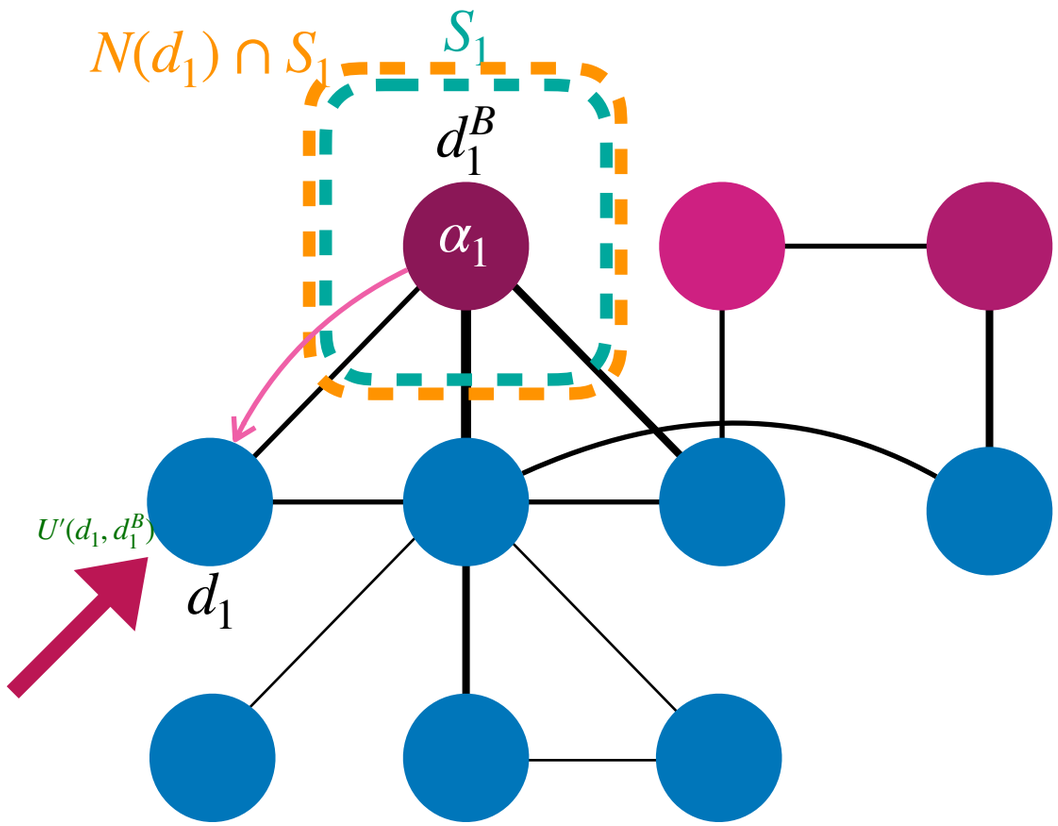

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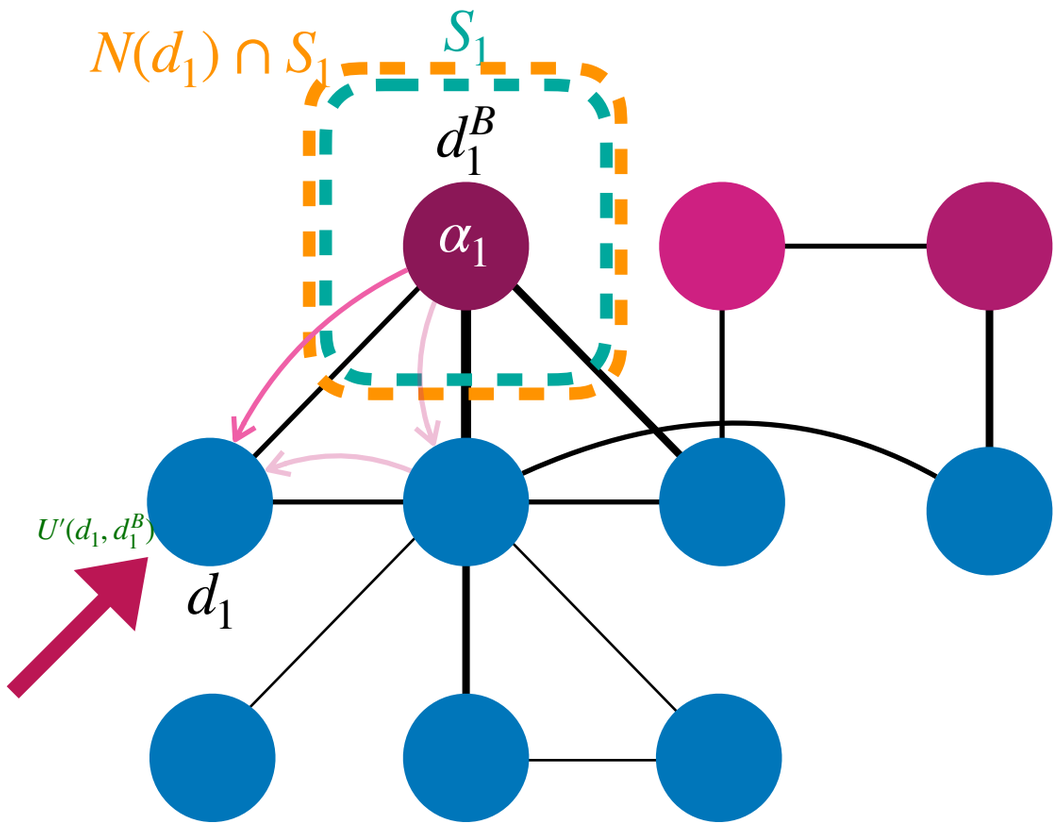

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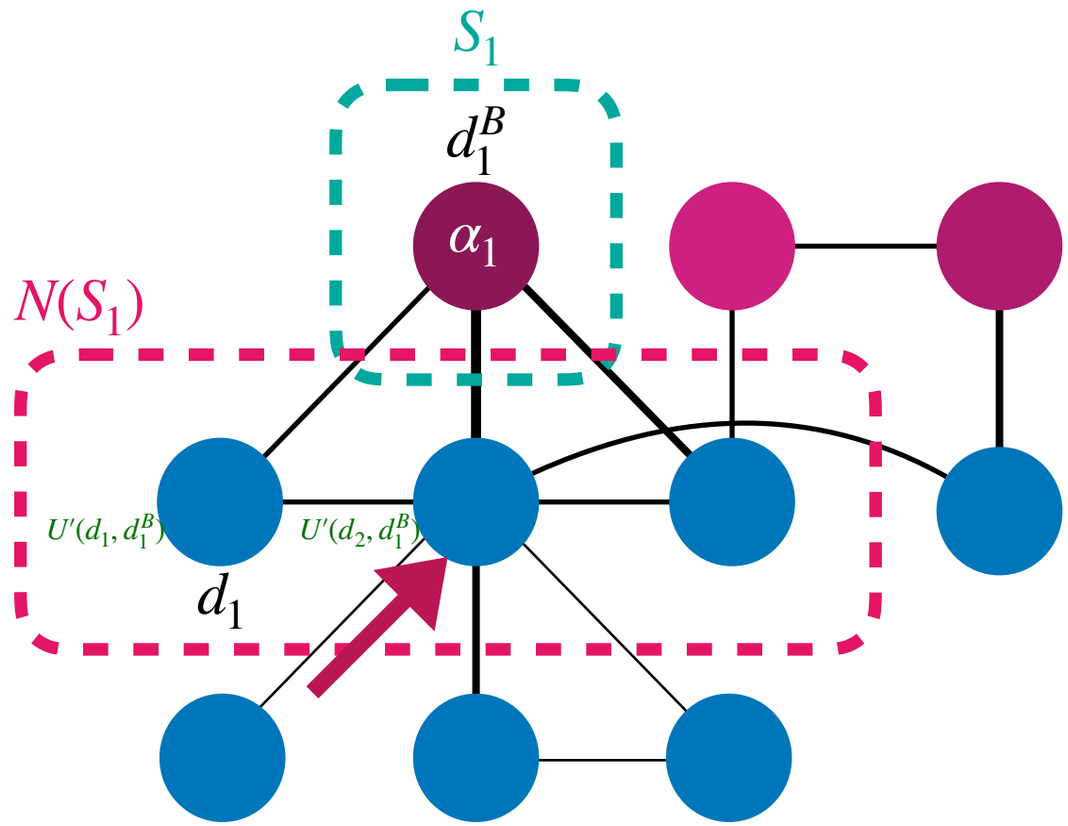

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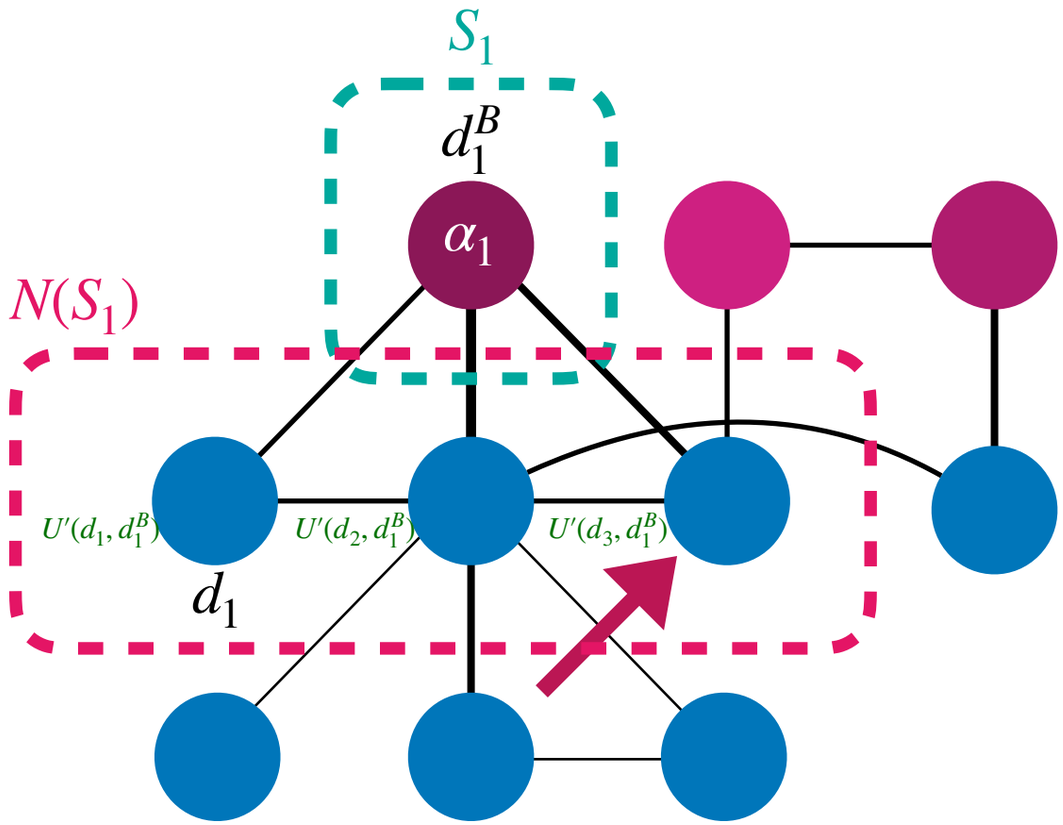

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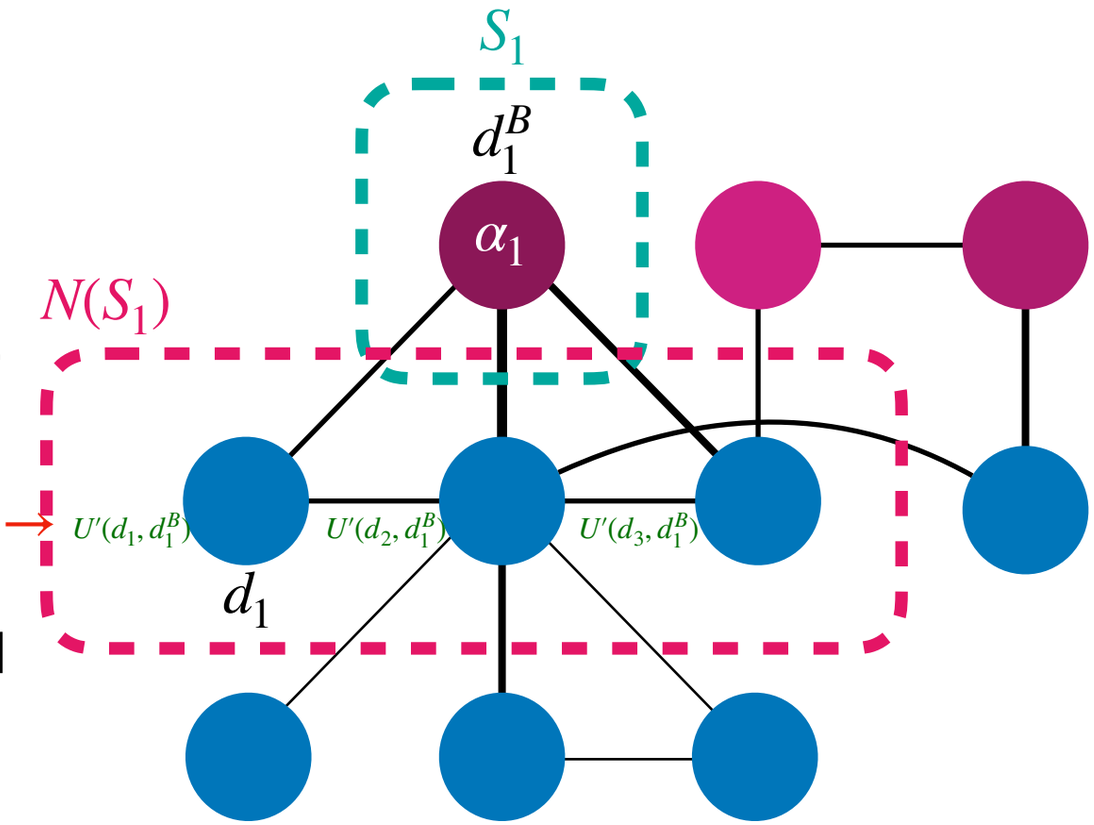

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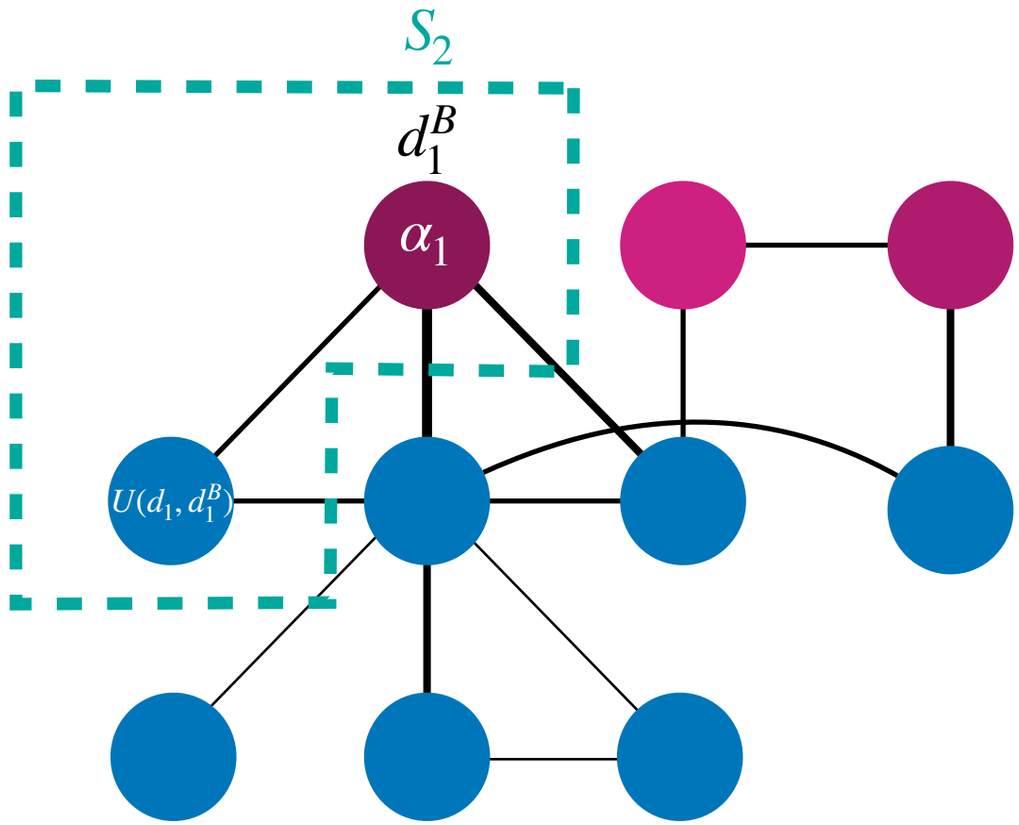
- Let $U(d, d')$ be the lowest upper bound on d imposed by d'.



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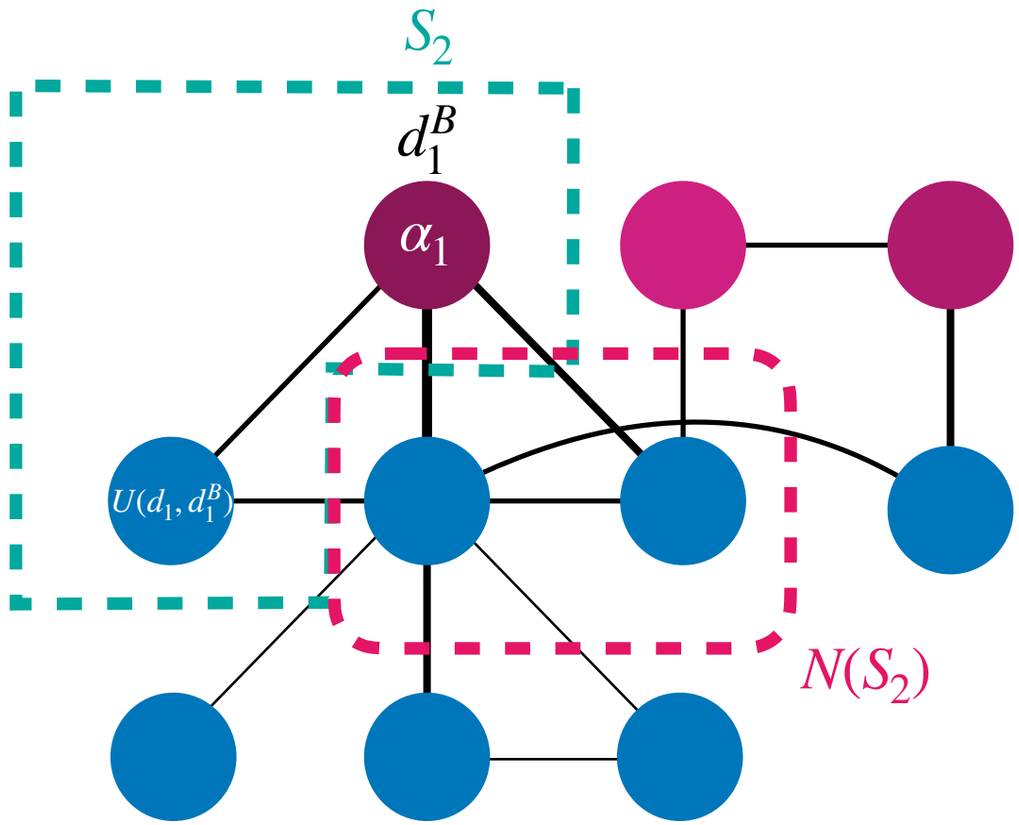

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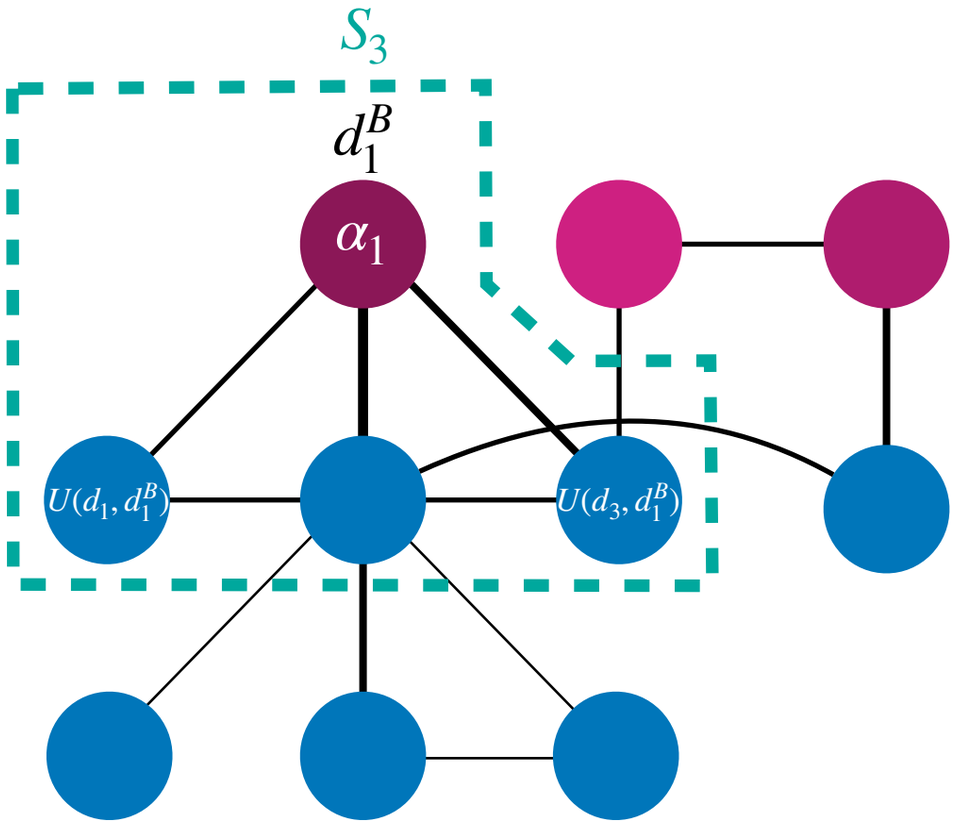

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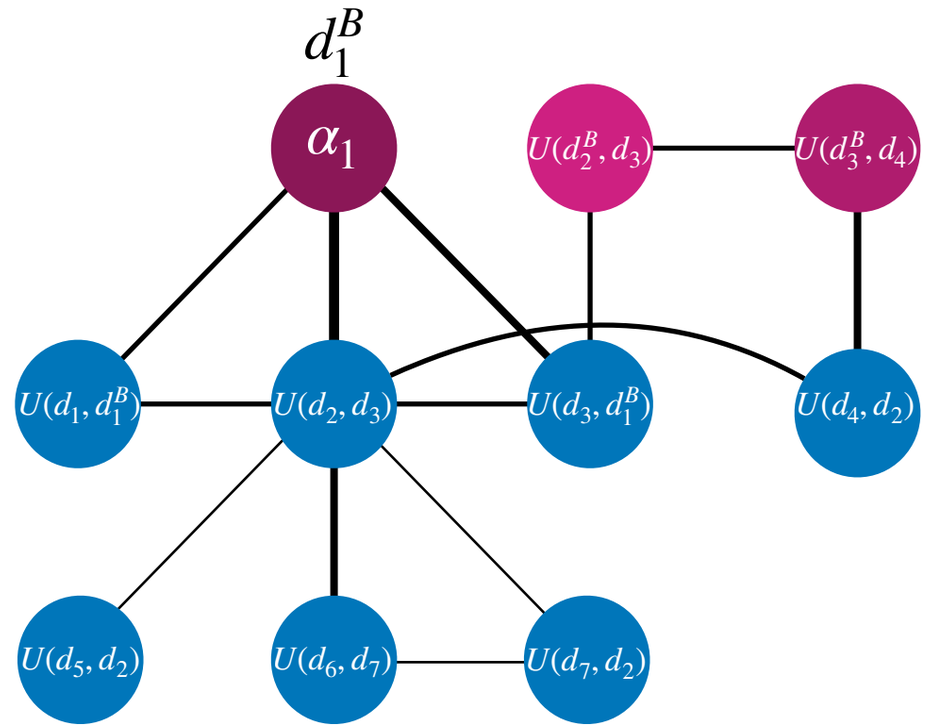

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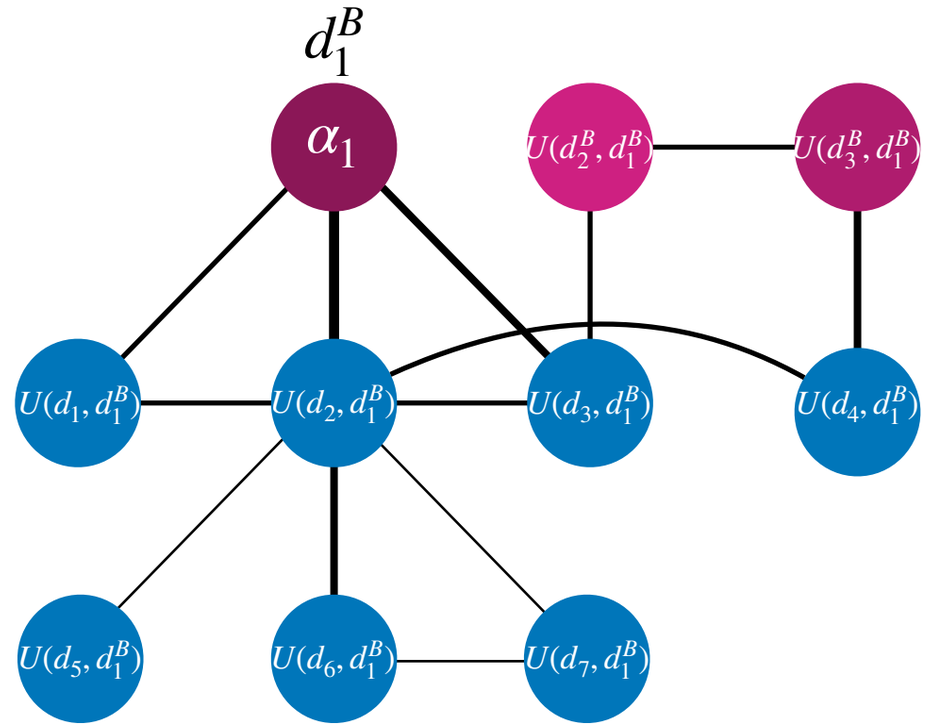
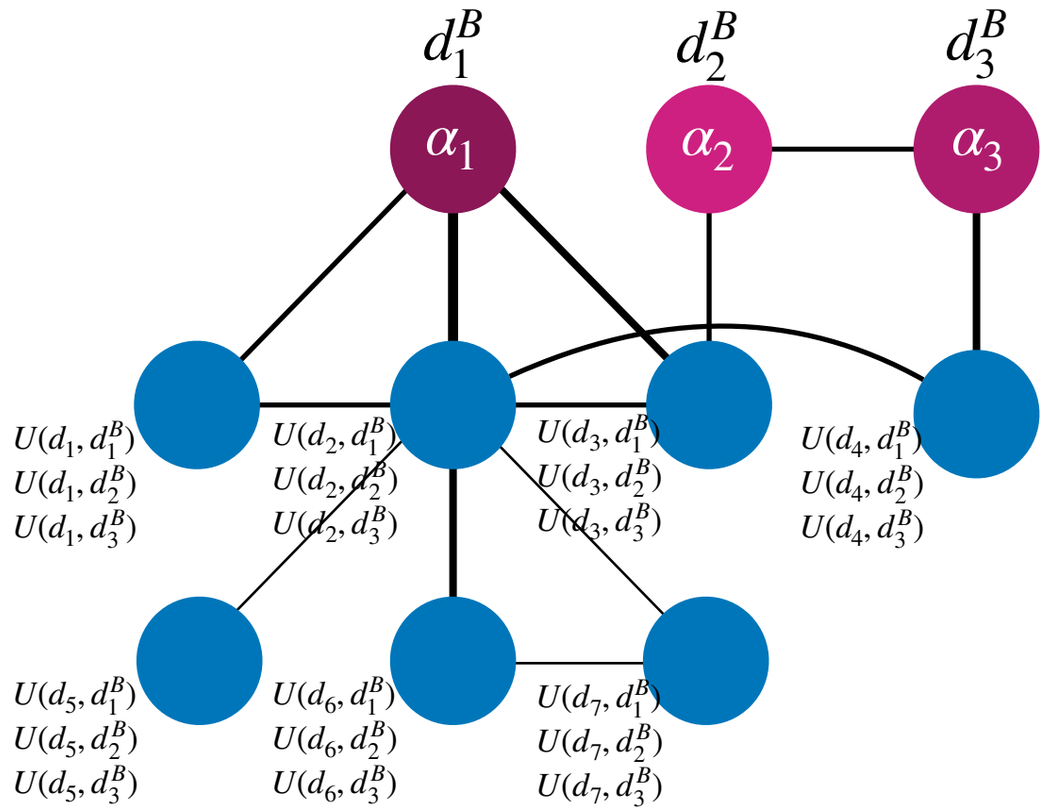


Algorithm

- Let $U(d)$ be the lowest upper bound on d over all the choices of d_i^B.



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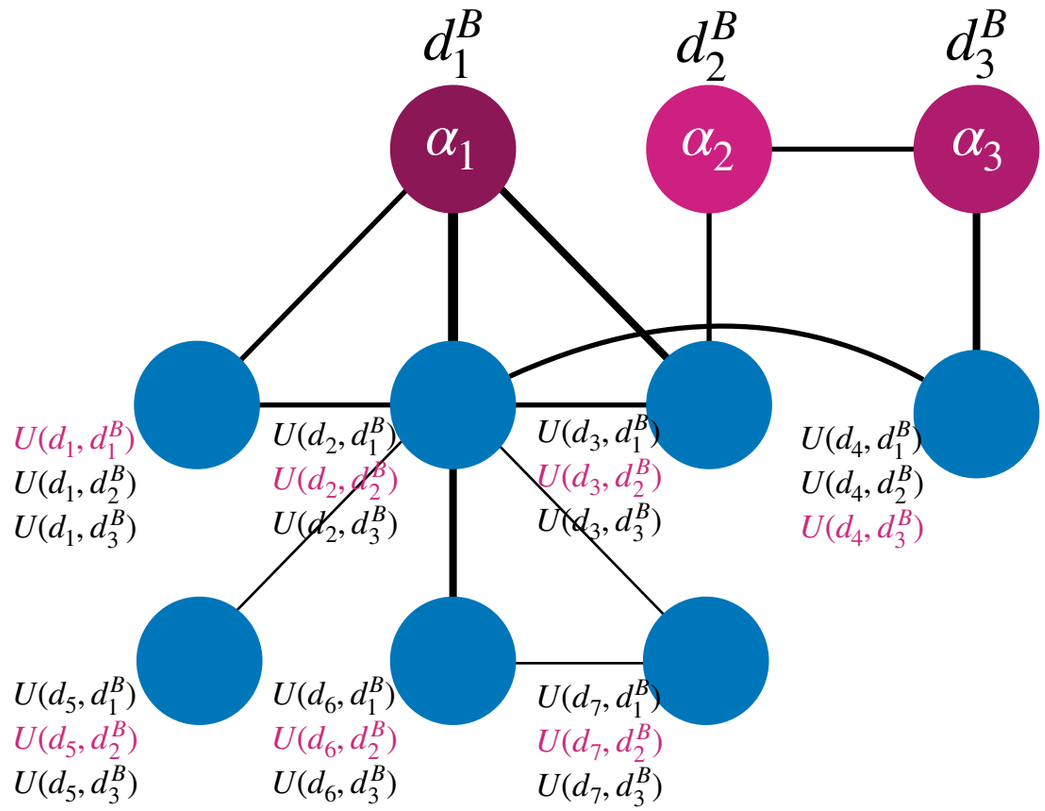

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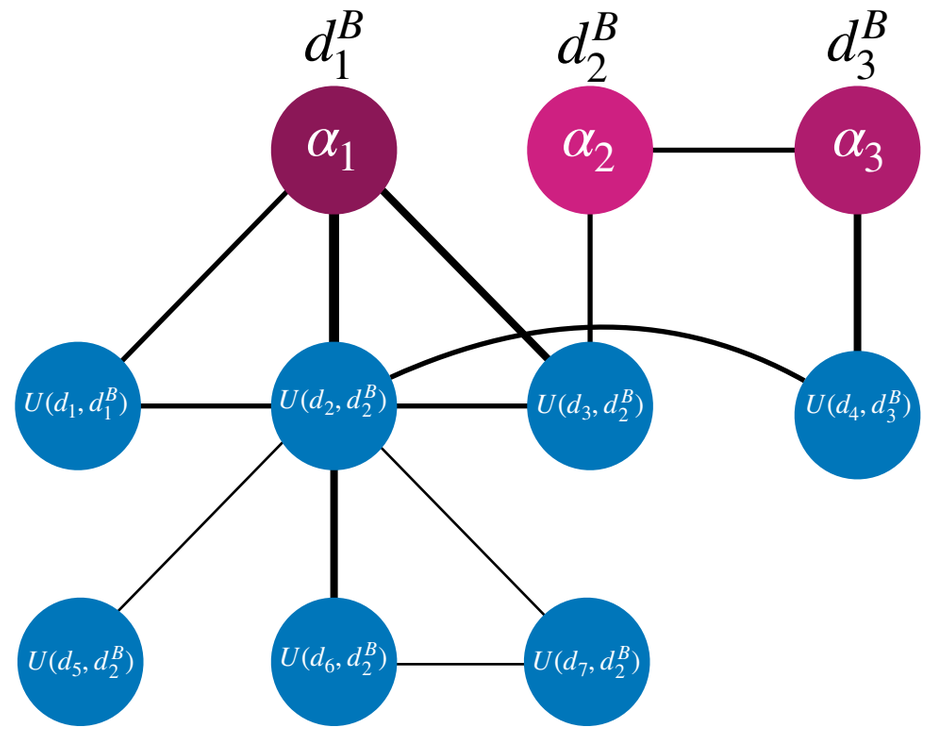

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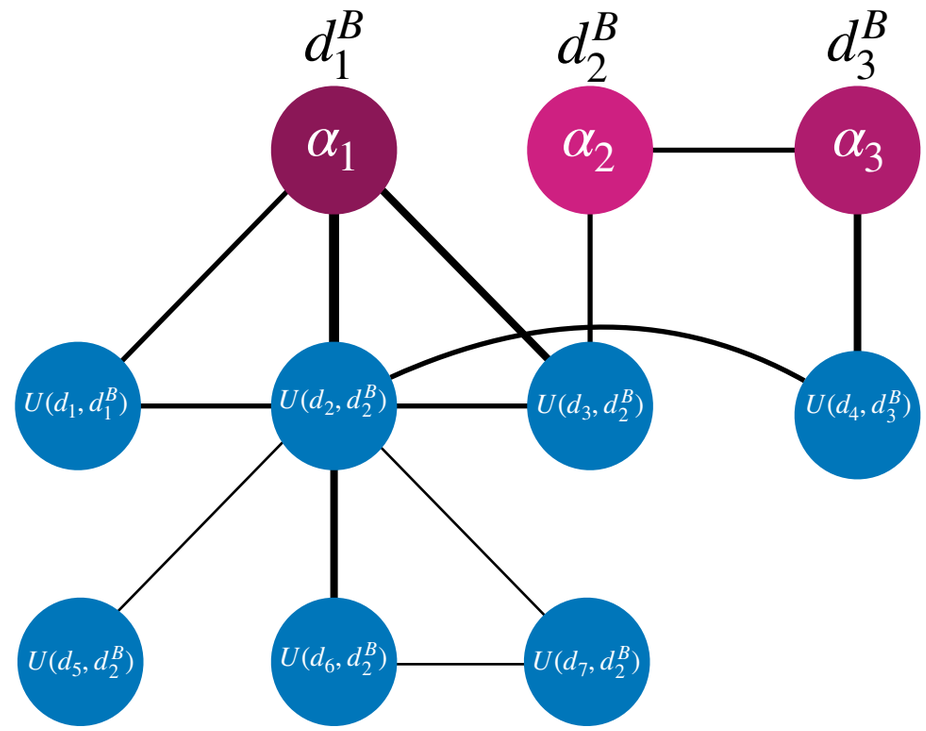
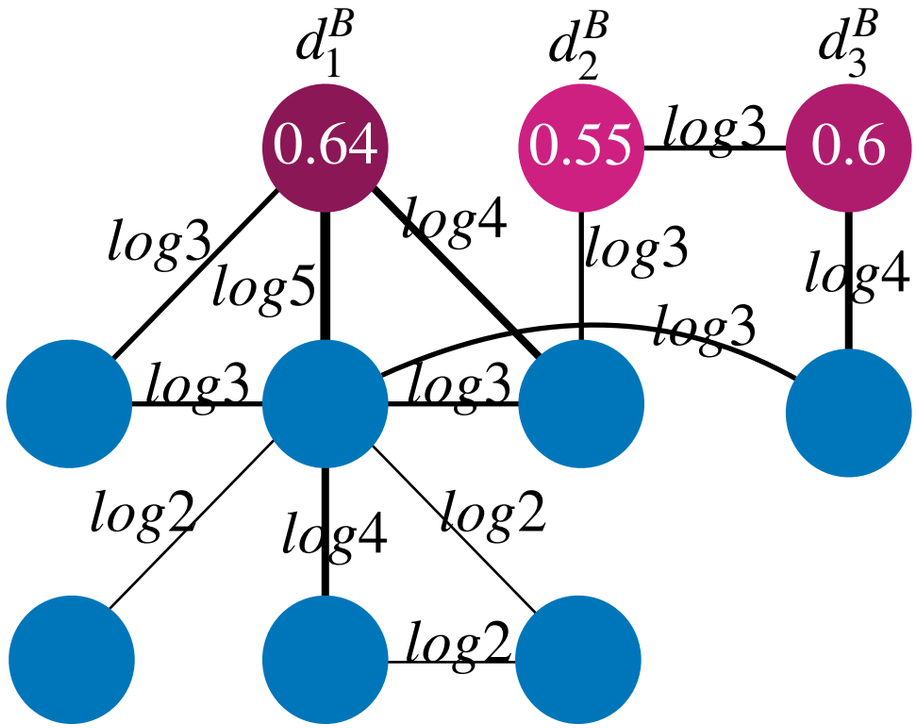


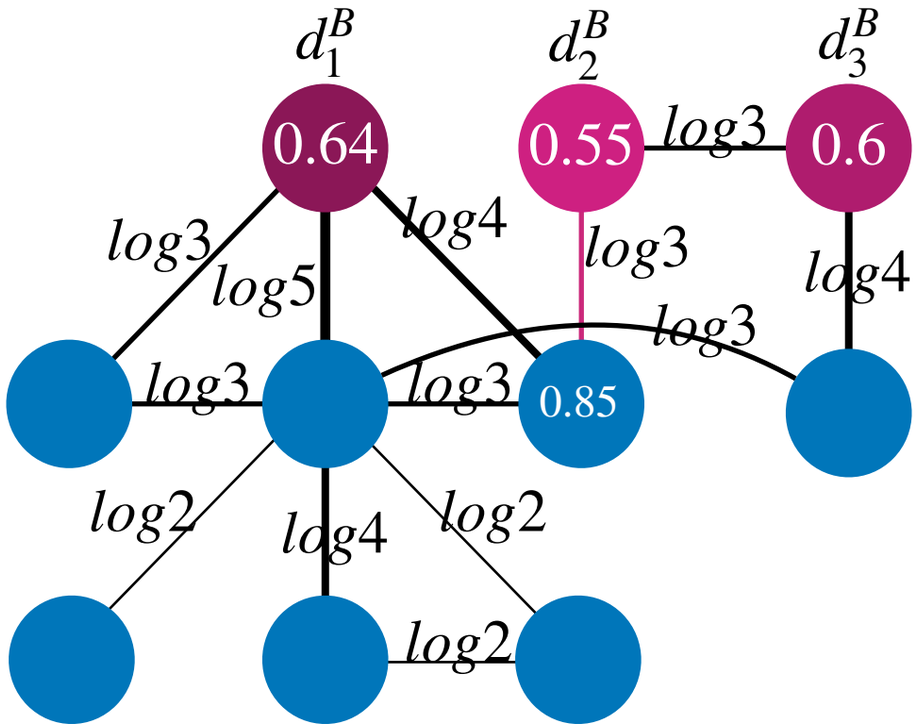
Theorem

The algorithm above finds the unique optimal heterogeneous DP mechanism in polynomial time, in the order of the number of vertices and edges.

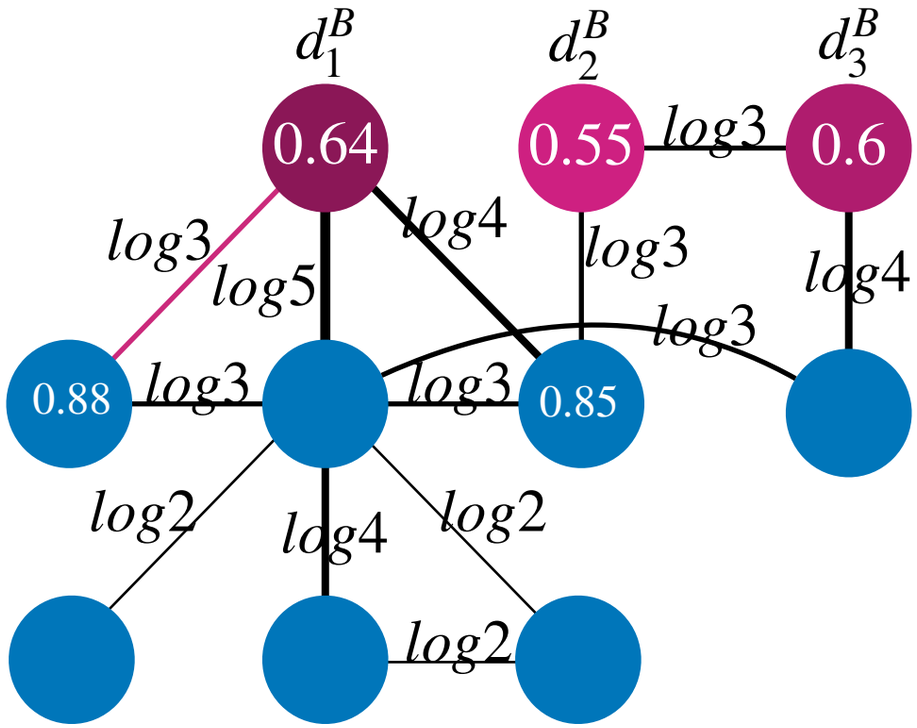
Example:


d_1^B
d_2^B
d_3^B
0.64
0.55
0.6
log3
log3
log4
log5
log3
log4
log3
log3
log3
log2
log4
log2
log2

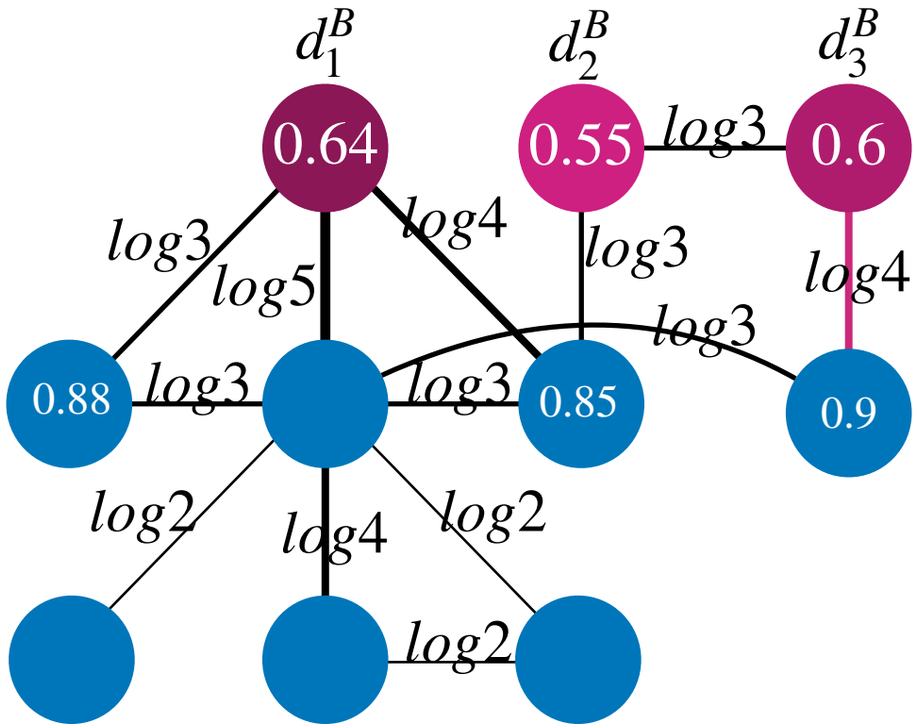
Example:


d_1^B
d_2^B
d_3^B
0.64
0.55
0.6
0.85
log3
log4
log3
log5
log3
log4
log3
log3
log3
log2
log4
log2
log2

Example:


d_1^B
d_2^B
d_3^B
0.64
0.55
0.6
0.88
0.85
log3
log4
log3
log5
log3
log3
log3
log4
log2
log4
log2
log2

Example:


d_1^B
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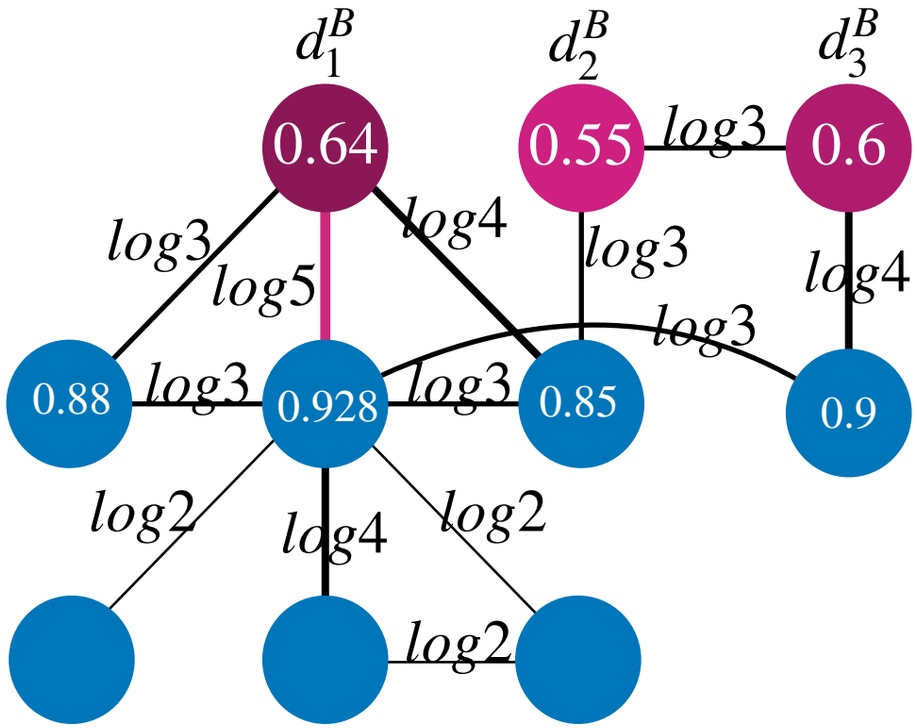
Example:


d_1^B
d_2^B
d_3^B
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0.85
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log2
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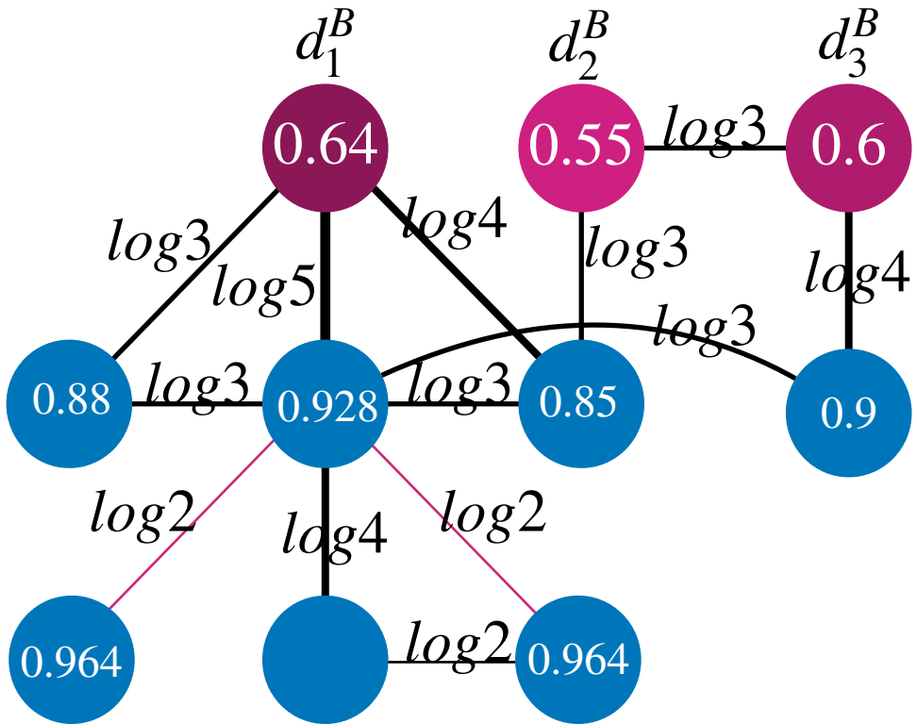
Example:


d_1^B
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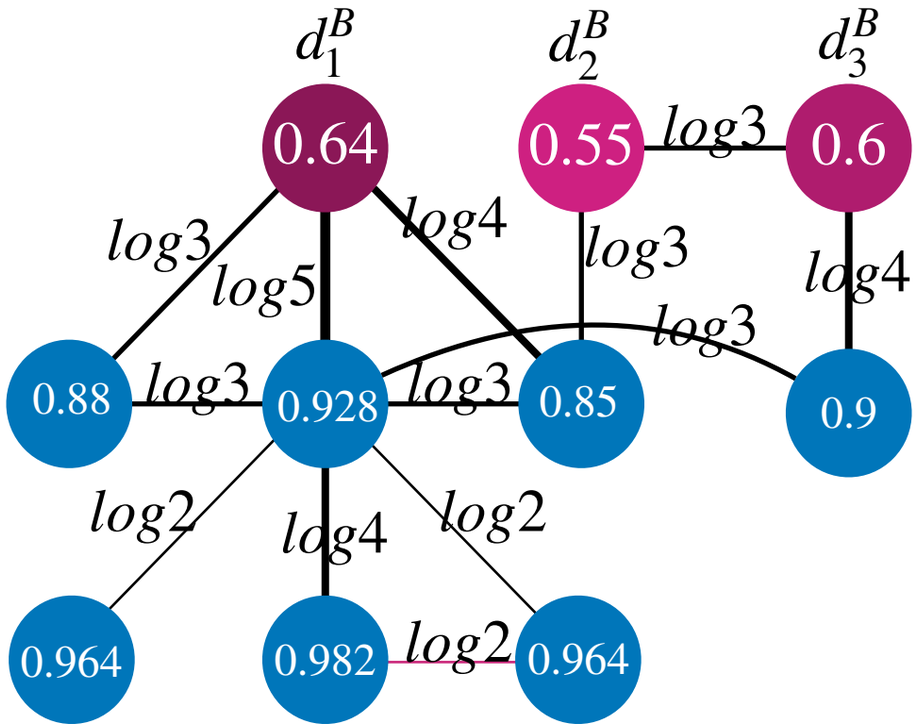
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log2
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log2
0.964
0.982
log2
0.964

Thank You
For Your Attention

Any Questions?